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Using Maximal Recurrence in Linear Threshold Competitive Layer Networks

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Abstract. We demonstrate the application of recent theoretical results on the stability of linear threshold (LT) networks to the competitive layer model architecture (CLM). LT networks can be efficiently built in silicon and exhibit the interesting behavior of coexistence of digital selection and analogue amplification in a single circuit. The CLM provides a large-scale network based on LT units, which was successfully applied to complex perceptual grouping tasks. We show that recent results on LT networks can be employed to operate the CLM with maximal recurrence close to its stability limits, causing strong contextual integration and improved grouping quality.

1 Introduction

A prominent model of the coupled dynamics of networks of neurons is the additive recurrent model

$$\dot{x}_i = -x_i + \sigma\Big(\sum_j w_{ij} x_j + h_i\Big), \tag{1}$$

with neuron activities $x_i \geq 0$, weights w_{ij}, external inputs h_i and transfer function σ. In recent years there has been a growing amount of literature on biologically-based models [2, 6, 1], where the transfer function is piecewise linear and non-saturating, given by $\sigma(x) = \max\big(0, k(x-\theta)\big)$. Threshold θ and gain $k > 0$ characterize this nonlinearity, denoted as semilinear or linear threshold (LT) function. It has been argued [3] that this transfer function is more appropriate than saturating nonlinearities, because cortical neurons rarely operate close to saturation, despite strong recurrent excitation. This indicates that the saturation may not be involved in the actual computations of the recurrent network, since the activation is dynamically bounded due to effective net interactions.

Recently Hahnloser et al.[4] demonstrated an efficient silicon design for LT networks and discussed the coexistence of analogue amplification and digital selection in their circuit. They showed that the stability analysis for symmetric LT networks can be reduced to the discussion of sets which contain the active neurons, where an active superset of an unstable attractor is also unstable and a subset of a stable attractor is also stable. They also stated a condition for non-divergence of symmetric systems, which, however, depends on eigenvalues of all possible subsystems, and is therefore difficult to evaluate for complex networks.

The coexistence of analog context-dependent activation and digital selection in an LT-circuit is achieved by a combination of avoiding runaway excitation and multistability. In the following we state two new conditions, proved in [7], which allow to estimate parameter bounds on the regime of maximal recurrent modulation in multistable LT networks. We then show how this can be employed for improved contextual integration in the CLM perceptual grouping model.

2 General LT Stability Conditions

The essential property that must be achieved to avoid runaway excitation is boundedness of the dynamics. Our first condition applies to arbitrary symmetric or non-symmetric LT networks described by (1).

Theorem 1. *If the self-coupling weights satisfy $w_{ii} < 1 - \sum_{j \neq i} \max(0, w_{ij})$ for all i, then the LT dynamics is bounded.*

The condition states that local inhibition with a strength corresponding to the sum of incoming excitatory weights is sufficient to avoid runaway excitation.

If the weights are symmetric with $w_{ij} = w_{ji}$ for all i, j, an additional criterion can be given that allows to improve the stability margins by taking into account also the non-local inhibitory contribution. The result is based on the energy function of the system which can be constructed for symmetric weights.

Theorem 2. *The LT dynamics is bounded if there exists a symmetric matrix $\hat{W}$ with $\hat{w}_{ij} \geq w_{ij}$ for all i, j, for which $\lambda_{\max}\{\hat{W}\} < 1$, where $\lambda_{\max}\{\hat{W}\}$ is the maximum eigenvalue of $\hat{W}$.*

This condition is different from the one given by Hahnloser et al. [4], which is based on considering submatrices for subsystems of the LT circuit. Note also the difference to the condition $\lambda_{\max}\{W\} < 1$ which would impose global asymptotic stability on the system, not allowing any multistability [7].

3 The CLM architecture

The CLM [5, 8] consists of a set of L layers of feature-selective neurons (see Fig. 1). The activity of a neuron at position r in layer α is denoted by $x_{r\alpha}$, and a *column* r denotes the set of the neuron activities $x_{r\alpha}$, $\alpha = 1, \dots, L$ that share a common position r. With each column a particular "feature" is associated, which is described by a set of parameters like e.g. local edge elements characterized by position and orientation (x_r, y_r, θ_r). A binding between two features, represented by columns r and r', is expressed by simultaneous activities $x_{r\hat{\alpha}} > 0$ and $x_{r'\hat{\alpha}} > 0$ that share a common layer $\hat{\alpha}$. All neurons in a column r are equally driven by an external input h_r which represents the significance of the detection of feature

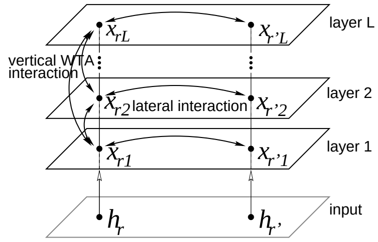

Fig. 1. The competitive layer model architecture (see text for description).

r by a preprocessing step. The afferent input h_r is fed to the activities $x_{r\alpha}$ with a connection weight $J > 0$.

Within each layer α the activities are coupled via the lateral connections $\mathsf{f}^{\alpha}_{rr'}$ which corresponds to the degree of compatibility between features r and r' and which is a symmetric function of the feature parameters, thus $\mathsf{f}^{\alpha}_{rr'} = \mathsf{f}^{\alpha}_{r'r}$. The purpose of the layered arrangement in the CLM is to enforce an assignment of the input features to the layers by the dynamics, using the contextual information stored in the lateral interactions. The unique assignment to a single layer is realized by a columnar Winner-Take-All (WTA) circuit, which uses mutual symmetric inhibitory interactions with absolute strength $J > 0$ between neural activities $x_{r\alpha}$ and $x_{r\beta}$ that share a common column r. Due to the WTA coupling, for a stable equilibrium state of the CLM only a neuron from one layer can be active within each column [8]. The number of layers does not predetermine the number of active groups for a particular run, since for sufficiently many layers only those are active that carry a salient group.

The combination of afferent inputs and lateral and vertical interactions is combined into the standard linear threshold additive activity dynamics

$$\dot{x_{r\alpha}} = -x_{r\alpha} + \sigma\Big(J(h_r - \sum_{\beta} x_{r\beta}) + \sum_{r'} \mathsf{f}^{\alpha}_{rr'} x_{r'\alpha} + x_{r\alpha}\Big) \qquad (2)$$

The CLM dynamics (2) is of the LT form as in (1), if a simple indexing correspondence is established with $i \leftrightarrow (r, \alpha)$. If we then apply Theorem 1, we obtain the following condition for the CLM:

Theorem 3. *The CLM dynamics is bounded and convergent if the condition* $J > \sum_{r' \neq r} \max(0, \mathsf{f}^{\alpha}_{rr'}) + \mathsf{f}^{\alpha}_{rr}$ *is satisfied.*

This can be derived by constructing the corresponding matrix of diagonal and nonpositive off-diagonal weights as in Theorem 1 for the whole CLM architecture. We note that this condition for the CLM was also derived in [8] using a different approach. The parameter J controls the influence of lateral contextual effects on the resulting activity pattern [8]. For J large compared to the lateral weights $\mathsf{f}^{\alpha}_{rr'}$, the single active neuron in a column reproduces its afferent input, $x_{r\alpha} \approx h_r$. For J small, the activity is modulated by the context of coactivated neurons in its layer with $x_{r\alpha} = h_r + J^{-1} \sum_{r'} \mathsf{f}^{\alpha}_{rr'} x_{r'\alpha}$. Therefore, if we choose J close to the stability bound of Theorem 3, the activity is strongly influenced by lateral effects. Theorem 2 gives a new, alternative condition for the CLM:

Theorem 4. *The CLM dynamics is bounded and convergent if $J > \lambda_{\max}\{\mathsf{f}^{\alpha}_{rr'}\}$ for all α, where $\lambda_{\max}\{\mathsf{f}^{\alpha}_{rr'}\}$ is the largest eigenvalue of the lateral interaction matrix within layer α.*

Proof. Construct a matrix $\hat{W}$ (see Theorem 2) by omitting all off-diagonal vertical interactions with $\hat{W}^{\alpha\beta}_{rr'} = -\delta_{rr'}\delta_{\alpha\beta}(J-1) + \delta_{\alpha\beta}\mathsf{f}^{\alpha}_{rr'}$. The Kronecker delta is defined by $\delta_{ij} = 1$ if $i = j$ and $\delta_{ij} = 0$ if $i \neq j$. The matrix $\hat{W}$ satisfies $\hat{W}^{\alpha\beta}_{rr'} \geq W^{\alpha\beta}_{rr'}$ for all r, r', α, β. Since $\hat{W}$ contains no cross-layer coupling it has a layer-wise block structure, where all entries corresponding to a coupling of neurons which are in different layers are zero. The eigenvalues of $\hat{W}$ are then given by the eigenvalues of the lateral interaction within layers, given by $\lambda_i\{\mathsf{f}^{\alpha}_{rr'}\} - J + 1$. Therefore, if $J > \lambda_{\max}\{\mathsf{f}^{\alpha}_{rr'}\}$, then $\lambda_{\max}\{\hat{W}\} < 1$ and the dynamics is bounded according to Theorem 2.

4 Results

We now apply the above stability conditions and investigate with quantitative measures the corresponding enhancement of grouping quality through strong contextual effects. We introduce two measuring functions to assess the quality of the final assignment state that is obtained from the CLM. Suppose, that for a synthetically generated feature set, each feature can be assigned in advance to one of M groups indexed by $l = 1, \ldots, M$. If l_r denotes the group index of feature r then the *label grouping measure* of an assignment state $\hat{\boldsymbol{\alpha}}$ is defined by

$$Q^l = \frac{1}{N^2}\sum_{rr'} q_{rr'}, \quad \text{where} \quad q_{rr'} = \begin{cases} 1 & \text{if } l_r = l_{r'} \text{ and } \hat{\alpha}(r) = \hat{\alpha}(r'), \\ 1 & \text{if } l_r \neq l_{r'} \text{ and } \hat{\alpha}(r) \neq \hat{\alpha}(r'), \\ 0 & \text{else .} \end{cases} \tag{3}$$

The quantity Q^l measures the proportion of correctly grouped pairs of features, if $Q^l = 1$ then all features are grouped in accordance with the a priori known object relations. If grouping on real images is performed, the object labels are generally not available. In that case the lateral contribution of the CLM energy function provides an indirect measure for the grouping quality. The *energy grouping measure* is defined by

$$Q^E = \sum_{\alpha}\sum_{rr'} \mathsf{f}^{\alpha}_{rr'} g(x_{r\alpha}) g(x_{r'\beta}), \quad \text{where} \quad g(x) = \begin{cases} 1 & \text{if } x > 0, \\ 0 & \text{if } x = 0. \end{cases} \tag{4}$$

The activities are passed through the squashing function $g(.)$ to make the value of Q^E independent of the contextual activity modulation, since here we are only interested in the binary group relations.

Theorems 3 and 4 give critical values for J, that characterizes the influence of recurrent modulation and lateral effects. For the CLM contour grouping setup, which we consider in the following, the eigenvalue bound of Theorem 4 results in a lower bound on J. Therefore we define $J_c = \max_{\alpha} \lambda_{\max}\{\mathsf{f}^{\alpha}_{rr}\}$ as the critical lower value for J and vary $\mu = J/J_c \geq 1$. The detailed lateral coupling scheme f^{α}_{rr}

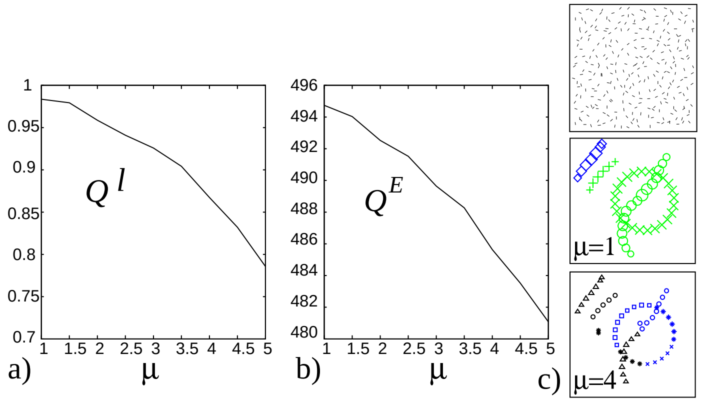

Fig. 2. Grouping performance dependent on contextual modulation for a synthetic image averaged over 100 simulations. a) shows the dependence of the labeling measure Q^l on μ. The energy grouping measure in b) shows an analog behavior with best results for $\mu \to 1$. c) Artificially generated feature set for contour grouping, for which the results in a) and b) were computed. Two grouping results for $\mu = 1$ and $\mu = 4$ are shown with symbols coding for different layers/groups. Symbol size represents activation, the background layer is omitted.

for all r, r', α for contour grouping with figure-ground segmentation is described in [8].

Figure 2 shows the dependency of Q^l and Q^E for the contour grouping of a synthetic pattern which consists of four objects and a noisy background. The energy measure Q^E varies in accordance with Q^l, which justifies its application in cases where Q^l is not available. The grouping quality increases by letting J approach the critical value J^c. This effect is mainly caused by the contextual destabilization of small fragments. Since larger fragments can develop stronger activity, they tend to attract small fragments into their group for $\mu \to 1$. The numerical simulation reveals that values $\mu < 1$ without instability are possible, since the conditions are sufficient and not necessary. We observed, however, that usually values less than $\mu < 0.95$ cause runaway instability, which indicates that the conditions are close to the maximal possible value for J. Figure 3 shows the corresponding results for a large feature set derived from a real image.

5 Discussion

Our results emphasize the flexibility of LT networks, if an application requires both digital selection for symbolic operations like assignment to a group/layer and analogue modulation for context integration. The new LT stability conditions allow to operate LT networks close to their stability limits, which causes strong dominance of recurrent activation over afferent external inputs. In the context of generation of orientation selectivity this has also been discussed as a

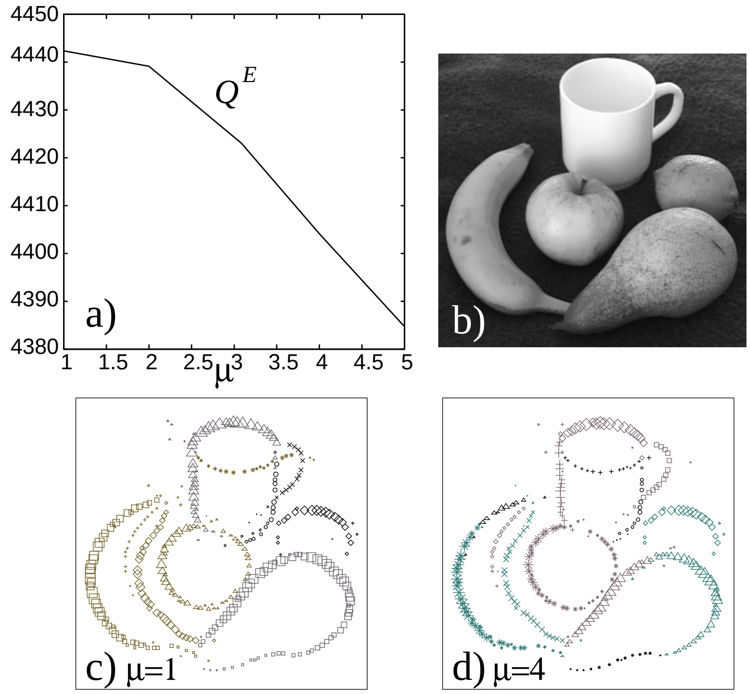

Fig. 3. a) Grouping quality Q^E dependent on contextual modulation for a real image shown in b) averaged over 50 simulations. The quality is enhanced by operating the CLM at $\mu \to 1$. c) and d) show the resulting groupings (background layer omitted).

symmetry-breaking process[2], which requires multistable systems. As the perceptual grouping applications of the CLM demonstrate, large-scale LT networks can provide powerful sensory processing architectures. They are also interesting for hardware implementations, if new approaches to their VLSI design [4] are applied to large-scale networks.

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