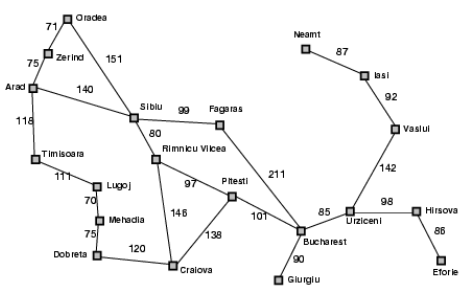
# Best-first search

- Idea: use an evaluation function *f(n)* for each node
  - estimate of "desirability"
  - → Expand most desirable unexpanded node
- Implementation:
  Order the nodes in fringe in decreasing order of desirability
- Special cases:
  - greedy best-first search
  - A* search

# Romania with step costs in km




| Straight-line distance to Bucharest | |
|---|---|
| Arad | 366 |
| Bucharest | 0 |
| Craiova | 160 |
| Dobreta | 242 |
| Eforie | 161 |
| Fagaras | 176 |
| Giurgiu | 77 |
| Hirsova | 151 |
| Iasi | 226 |
| Lugoj | 244 |
| Mehadia | 241 |
| Neamt | 234 |
| Oradea | 380 |
| Pitesti | 10 |
| Rimnicu Vilcea | 193 |
| Sibiu | 253 |
| Timisoara | 329 |
| Urziceni | 80 |
| Vaslui | 199 |
| Zerind | 374 |

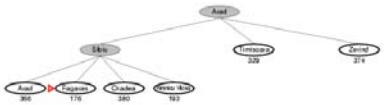
# Greedy best-first search

- Evaluation function *f(n)* = *h(n)* (heuristic)
- = estimate of cost from *n* to *goal*
- e.g., $h_{SLD}(n)$ = straight-line distance from *n* to Bucharest
- Greedy best-first search expands the node that appears to be closest to goal

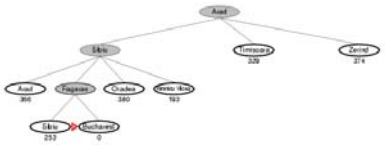
# Greedy best-first search example

# Greedy best-first search example

# Greedy best-first search example



# Greedy best-first search example



# Properties of greedy best-first search

- Complete? No – can get stuck in loops, e.g., Iasi → Neamt → Iasi → Neamt → □
- Time? $O(b^m)$, but a good heuristic can give dramatic improvement□
- Space? $O(b^m)$ -- keeps all nodes in memory□
- Optimal? No□

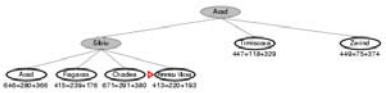
# A* search

- Idea: avoid expanding paths that are already expensive□
- Evaluation function $f(n) = g(n) + h(n)$□
- $g(n)$ = cost so far to reach $n$
- $h(n)$ = estimated cost from $n$ to goal
- $f(n)$ = estimated total cost of path through $n$ to goal□

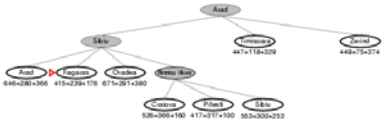
# A* search example

# A* search example

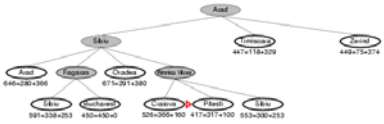
# A* search example



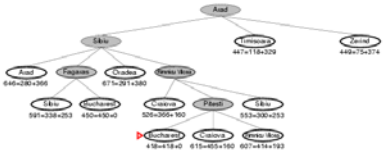
# A* search example



# A* search example



# A* search example

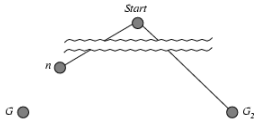


# Admissible heuristics

- A heuristic *h(n)* is admissible if for every node *n*, $h(n) \le h^*(n)$, where $h^*(n)$ is the true cost to reach the goal state from *n*.
- An admissible heuristic never overestimates the cost to reach the goal, i.e., it is optimistic
- Example: $h_{SLD}(n)$ (never overestimates the actual road distance)
- Theorem: If *h(n)* is admissible, A* using `TREE-SEARCH` is optimal

# Optimality of A* (proof)

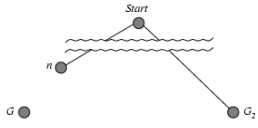
- Suppose some suboptimal goal $G_2$ has been generated and is in the fringe. Let *n* be an unexpanded node in the fringe such that *n* is on a shortest path to an optimal goal *G*.




- $f(G_2) = g(G_2)$ since $h(G_2) = 0$
- $g(G_2) > g(G)$ since $G_2$ is suboptimal
- $f(G) = g(G)$ since $h(G) = 0$
- $f(G_2) > f(G)$ from above

# Optimality of A* (proof)

- Suppose some suboptimal goal $G_2$ has been generated and is in the fringe. Let *n* be an unexpanded node in the fringe such that *n* is on a shortest path to an optimal goal *G*.



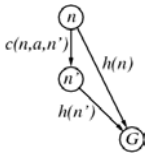

- $f(G_2)$ > f(G) from above
- h(n) ≤ h^*(n) since h is admissible
- g(n) + h(n) ≤ g(n) + h*(n)
- f(n) ≤ f(G)

Hence $f(G_2) > f(n)$, and A* will never select $G_2$ for expansion

# Consistent heuristics

- A heuristic is consistent if for every node *n*, every successor *n'* of *n* generated by any action *a*,

*h(n) ≤ c(n,a,n') + h(n')*



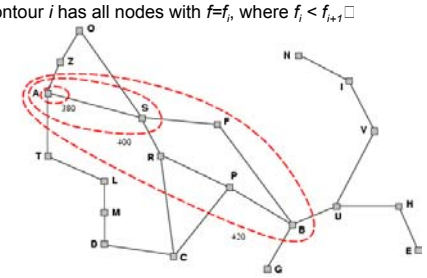

- If *h* is consistent, we have

f(n') = g(n') + h(n')
= g(n) + c(n,a,n') + h(n')
≥ g(n) + h(n)
= f(n)

- i.e., *f(n)* is non-decreasing along any path.
- Theorem: If *h(n)* is consistent, A* using GRAPH-SEARCH is optimal

# Optimality of A*

- A* expands nodes in order of increasing *f* value
- Gradually adds "*f*-contours" of nodes
- Contour *i* has all nodes with $f=f_i$, where $f_i < f_{i+1}$




# Properties of A*

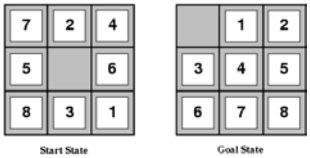
- Complete? Yes (unless there are infinitely many nodes with f ≤ *f(G)* )
- Time? Exponential
- Space? Keeps all nodes in memory
- Optimal? Yes

# Admissible heuristics

E.g., for the 8-puzzle:

- $h_1(n)$ = number of misplaced tiles
- $h_2(n)$ = total Manhattan distance

(i.e., no. of squares from desired location of each tile)



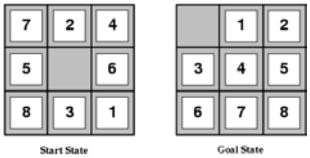

- $h_1(S)$ = ?
- $h_2(S)$ = ?

# Admissible heuristics

E.g., for the 8-puzzle:

- $h_1(n)$ = number of misplaced tiles
- $h_2(n)$ = total Manhattan distance

(i.e., no. of squares from desired location of each tile)




- $h_1(S)$ = ? 8
- $h_2(S)$ = ? 3+1+2+2+2+3+3+2 = 18

# Dominance

- If $h_2(n) \geq h_1(n)$ for all *n* (both admissible)
- then $h_2$ dominates $h_1$
- $h_2$ is better for search

- Typical search costs (average number of nodes expanded):

- *d=12* IDS = 3,644,035 nodes
  $A^*(h_1)$ = 227 nodes
  $A^*(h_2)$ = 73 nodes
- *d=24* IDS = too many nodes
  $A^*(h_1)$ = 39,135 nodes
  $A^*(h_2)$ = 1,641 nodes