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Realtime 3D Computer Graphics
Virtual Reality

Viewing and projection

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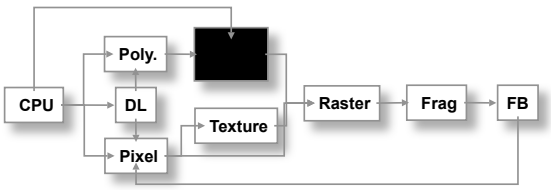
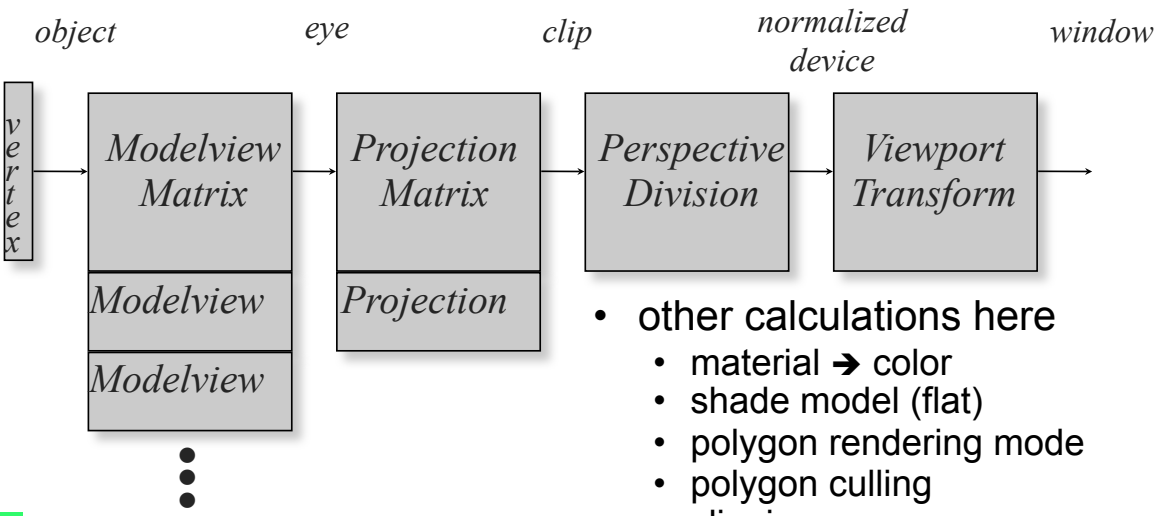
Classical and General Viewing

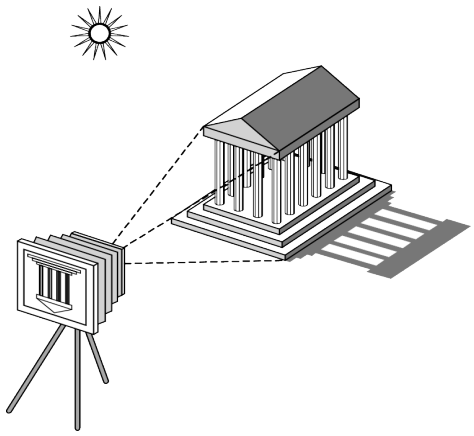
Transformation Pipeline





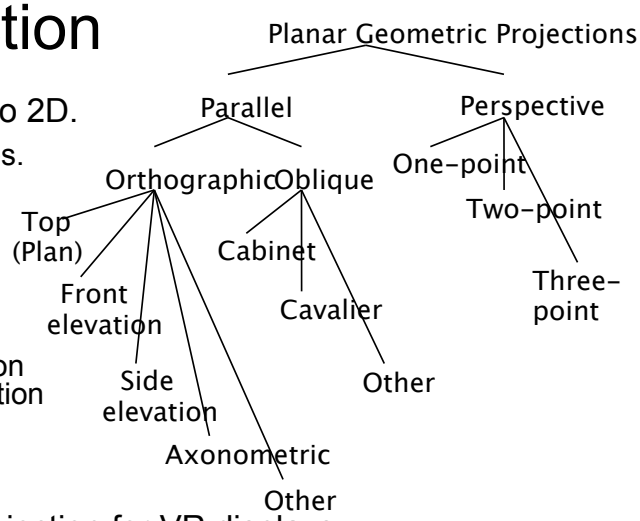
- other calculations here
  - material ➔ color
  - shade model (flat)
  - polygon rendering mode
  - polygon culling
  - clipping

Viewing



- Process for “Seeing” a world
- Projection of a 3D world onto a 2D plane
- Synthetic Camera Model
  - Location of viewer and view plane
  - What can be seen (Culling and Clipping)
  - How relationships are maintained
    - Parallel Lines
    - Angles
    - Distances (Foreshortening)
    - Relation to Viewer
  - Objects vs. Scenes:
    - Some viewing techniques better suited for viewing single objects rather than entire scenes
    - Viewing an object from the outside (external viewing)
      - Engineering, External Buildings, …
    - Viewing an object from within (internal viewing)
      - Internal Buildings, Games, …

Viewing and projection

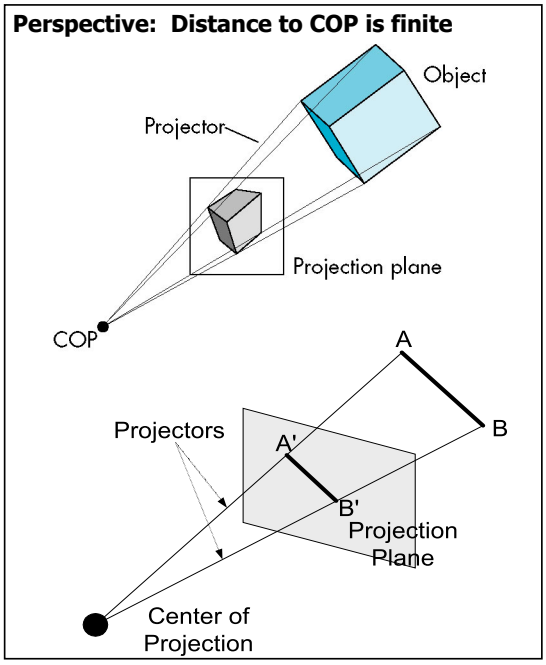


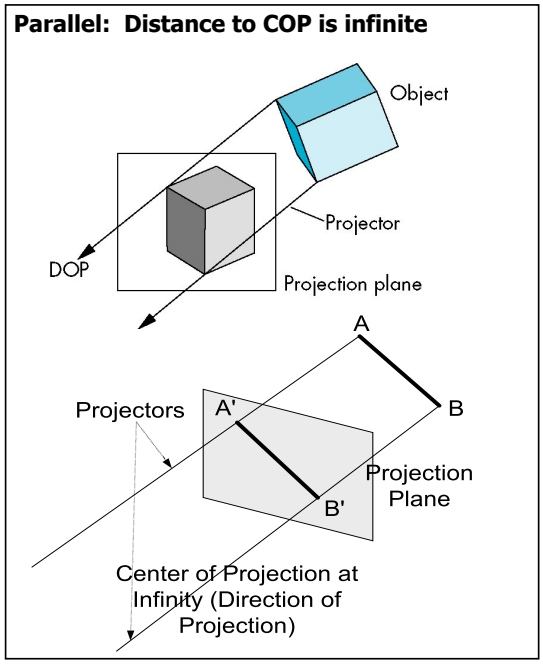
- Map transformed world from 3D to 2D.
  - by appropriate projection matrices.
- Several projection types exist:
- Perspective projection is of major interest for 3D-CG.
- Parallel projection importance
  - Pipeline often separates projection normalization from 3D-2D projection for additional depth testing.
- Calculation of the perspective projection for VR displays:
  - Display often require off-axis projection.
    - COPs are constantly moving.
    - Projection calculation has to be performed for 2 eyes.
  - Image planes might not be flat.

Definitions

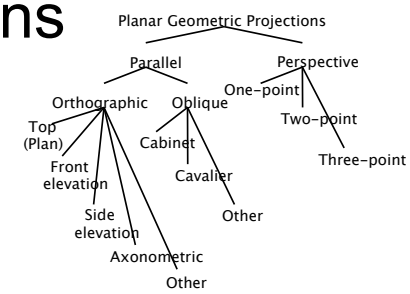
- ***Projection***: a transformation that maps from a higher dimensional space to a lower dimensional space (e.g. 3D->2D)
- ***Center of projection*** **(COP):** the position of the eye or camera with respect to which the projection is performed (also eye point, camera point, proj. reference point)
- **Direction of projection (DOP):** the direction of an eye or camera assumed to be infinite far away.
- ***Projection plane***: in a 3D->2D projection, the plane to which the projection is performed (also view plane)
- **Projectors:** lines from coordinate in original space to coordinate in projected space

Projections

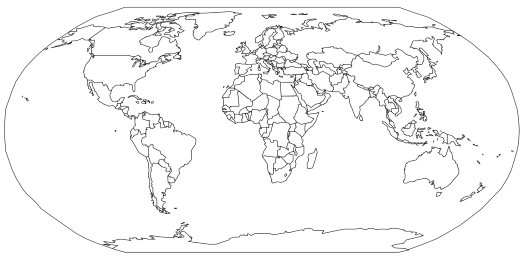




Planar Geometric Projections



- Projection onto a plane
- Projectors are straight lines
- Alternatives:
  - Some Cartographic Projections
  - Omnimax

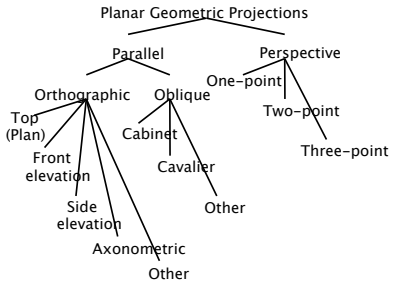


Parallel Projections

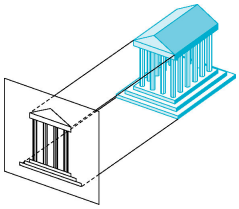
- Orthographic: Direction of projection is orthogonal to the projection plane
  - Elevations: Projection plane is perpendicular to a principal axis
    - Front
    - Top (Plan)
    - Side
  - Axonometric: Projection plane is not orthogonal to a principal axis
  - Isometric: Direction of projection makes equal angles with each principal axis.

- Oblique: Direction of projection is not orthogonal to the projection plane; projection plane is normal to a principal axis
  - Cavalier: Direction of projection makes a 45° angle with the projection plane
  - Cabinet: Direction of projection makes a 63.4° angle with the projection plane

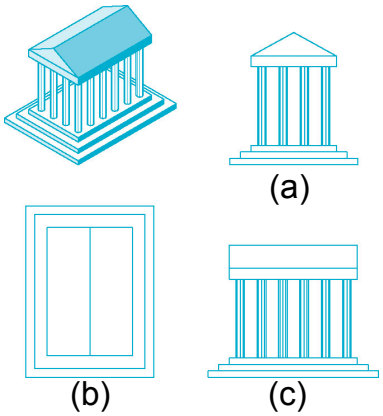
Orthographic Projections



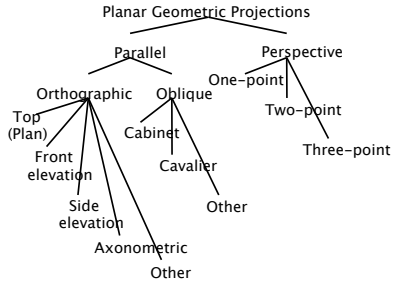
- "Special case" of perspective projection
- Parallel projectors perpendicular to projection plane



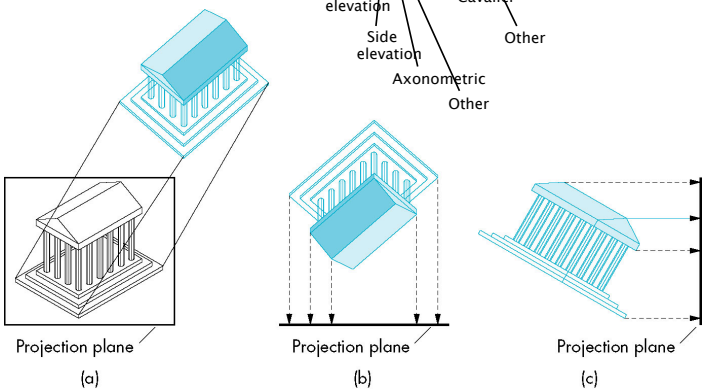
- Multiview
  - Projection Plane Parallel to Principle Faces
  - Classical Drafting Views
  - Preserves both distance and angles
  - Suitable to Object Views, not scenes
  - (a): Front-Elevation
  - (b): Top or Plan-Elevation
  - (c): Side-Elevation

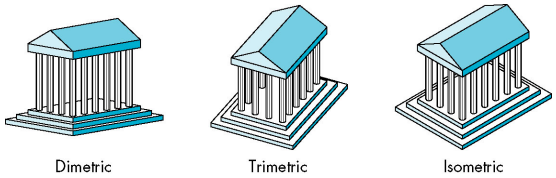


Axonometric Projections



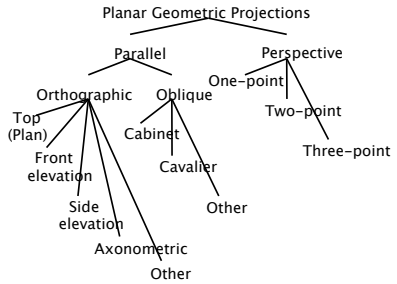
- Projection plane can have any orientation to object
- Parallel lines preserved
- Angles are not preserved
- Foreshortening:
  - Length is shorter in image space than in object space
  - Uniform Foreshortening (Perspective projections: foreshortening is dependent on distance from object to COP)



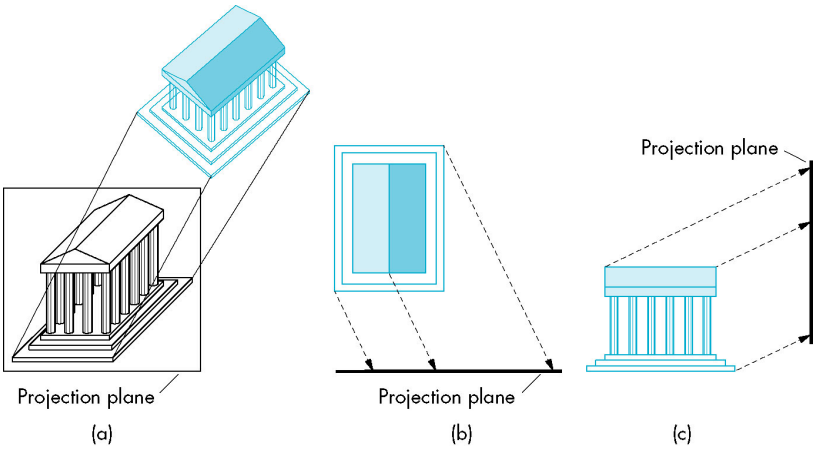


- Isometric: Symmetric to three faces
- Dimetric: Symmetric to two faces
- Trimetric: General Axonometric case

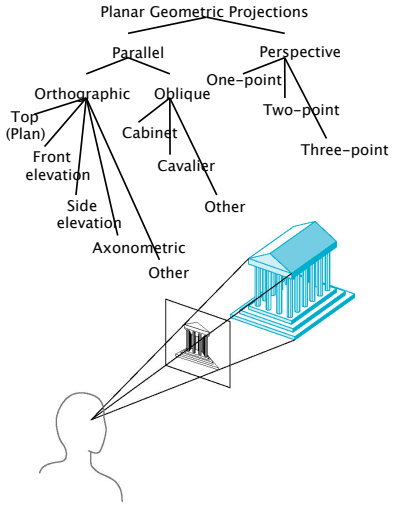
Oblique Projections



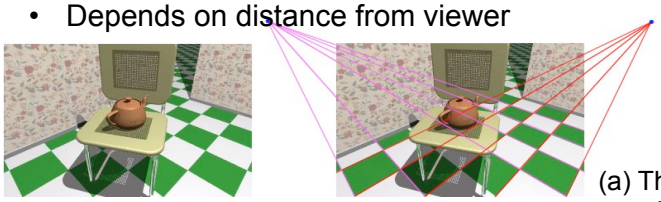
- Parallel Projections not perpendicular to projection plane
- Oblique projection types:
  - Cavalier: 45-degree Angles from Projection Plane
  - Cabinet: Arctan(2) or 63.4-degree Angles from Projection Plane



Perspective Projections



- First discovered by Donatello, Brunelleschi, and DaVinci during Renaissance
- Parallel lines appear to converge to single point
- Foreshortening:
  - Objects closer to viewer look larger
  - Length is not preserved
  - Depends on distance from viewer



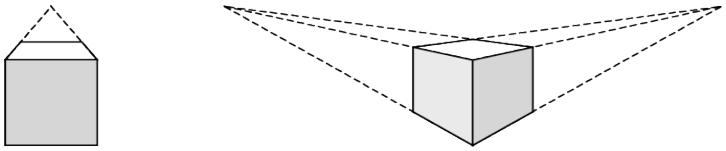
(a) (b) (c)

(a) Three-point:
Three principal axes cut by projection plane
Three axis vanishing points

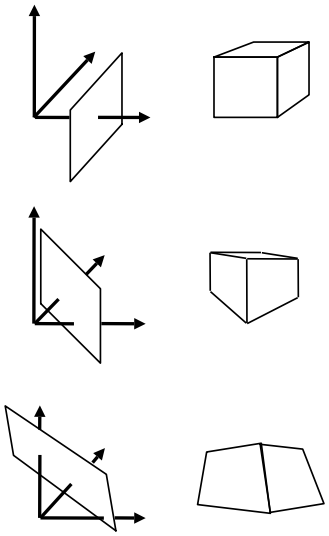
(b) Two-point:
Two principal axes cut by projection plane
Two axis vanishing points

(c) One-point:
One principal axis cut by projection plane
One axis vanishing point

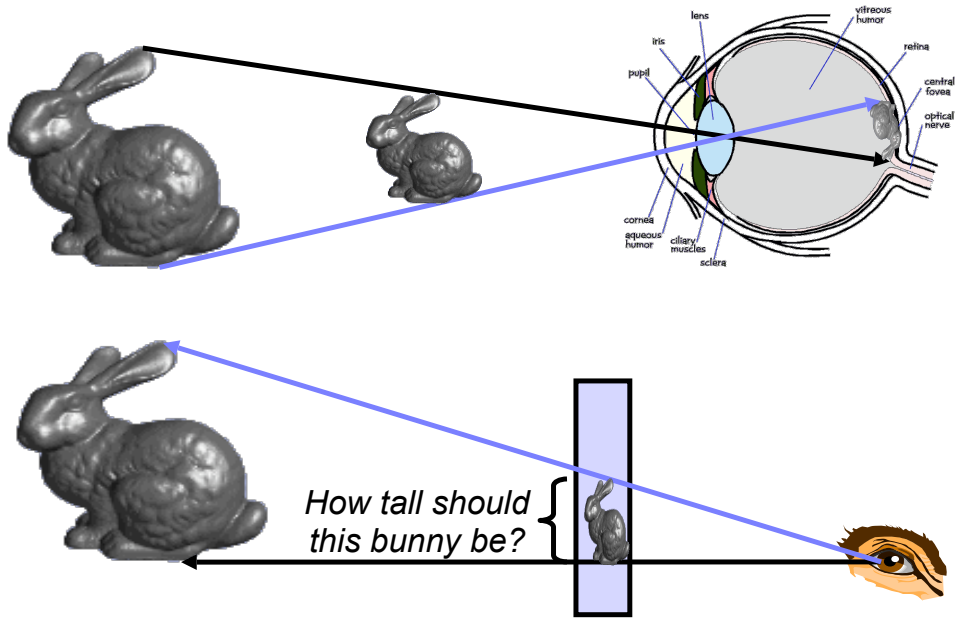
Vanishing Points



- Perspective Projection of any set of parallel line (not perpendicular to the projection plane) converge to a *vanishing point*
- Infinity of vanishing points
  - one of each set of parallel lines
- Axis Vanishing Points
  - Vanishing point of lines parallel to one of the three principal axes
  - There is one axis vanishing point for each axis cut by the projection plane
  - At most, 3 such points
  - Perspective Projections are categorized by number of axis vanishing points



Perspective Foreshortening



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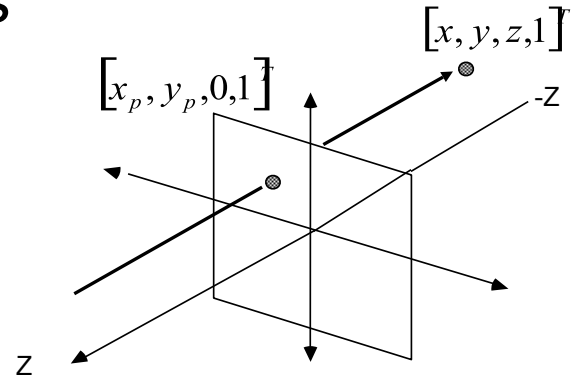
Projection Transformations

NOTE: Throughout the following discussions, we assume a camera coordinate system with COP at the origin and DOP along the z axis (OpenGL-like). The concepts are the same for any arbitrary viewing configuration.

Orthogonal Projections

- DOP parallel to z-axis
- Looking down negative z
- Display Plane at z=0
- Special Case of Perspective Projection



$$x_p = x,$$
$$y_p = y,$$
$$z_p = 0$$

$$p_{orth} = M_{orth} p = \begin{bmatrix} x_p \\ y_p \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

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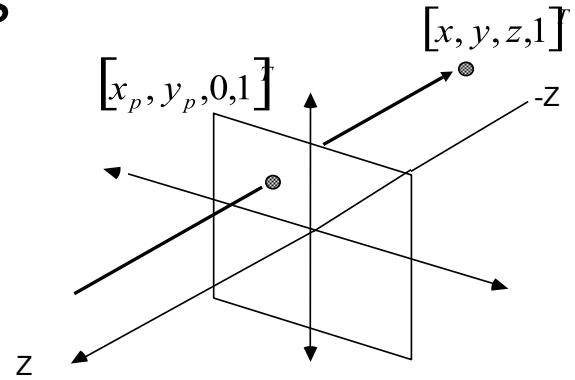

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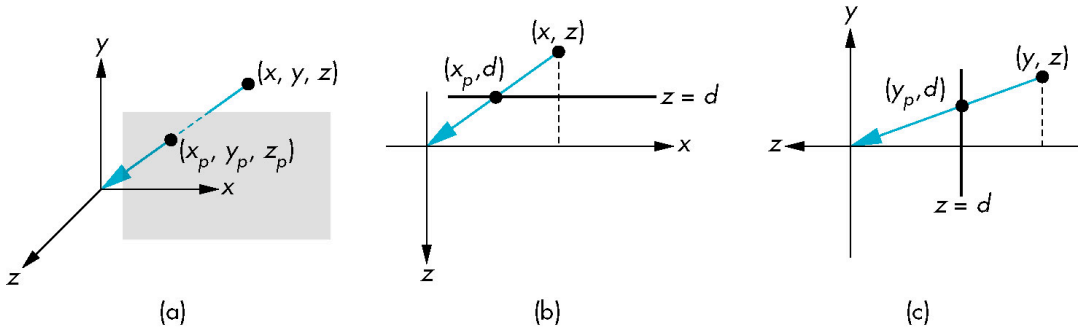
Perspective Projection: Points onto a Plane

- Projection of world onto display plane involves a perspective transformation:
  - $p' = M_{per} p$



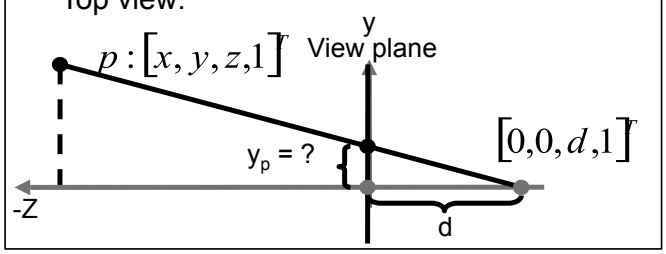
- Observe: $(b): \frac{x}{z} = \frac{x_p}{d} \Leftrightarrow x_p = \frac{x}{z/d}$ $\quad (c): \frac{y}{z} = \frac{y_p}{d} \Leftrightarrow y_p = \frac{y}{z/d}$

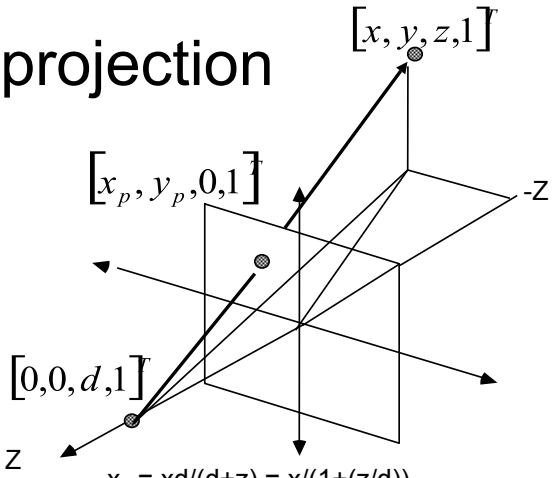
➡ Results in non-uniform foreshortening, **not** affine, **not** reversible

One-point perspective projection

- Center of Projection on the poitive z-axis
- Viewplane in the x-y plane.

➢ Geometry of similar triangles.
Top view:



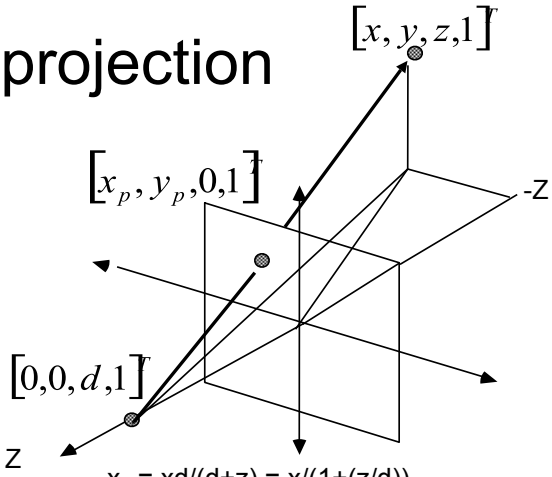


$x_p = xd/(d+z) = x/(1+(z/d))$
$y_p = yd/(d+z) = y/(1+(z/d))$
$z_p = 0$

$$p_{per} = M_{per} p = \begin{bmatrix} ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} ? \\ ? \\ ? \\ ? \end{bmatrix} = \begin{bmatrix} x/(1+(z/d)) \\ y/(1+(z/d)) \\ 0 \\ 1 \end{bmatrix}$$

One-point perspective projection

- Center of Projection on the positive z-axis
- Viewplane in the x-y plane.



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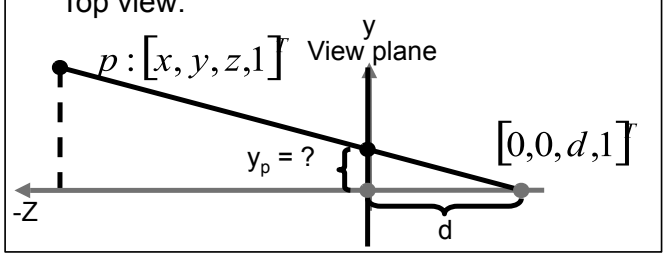

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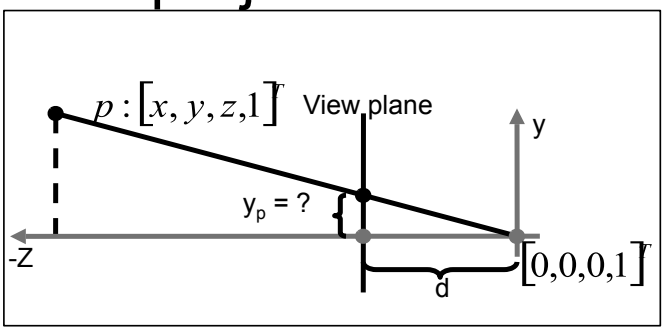
One-point perspective projection

- Center of Projection at origin
- Viewplane **not** in the x-y plane.

$$z_p = d$$
$$\frac{x}{z} = \frac{x_p}{d} \Leftrightarrow x_p = \frac{x}{z/d}$$
$$\frac{y}{z} = \frac{y_p}{d} \Leftrightarrow y_p = \frac{y}{z/d}$$



- Point p in 3D become lines through origin in 4D: $p = [wx \quad wy \quad wz \quad w]^T$

➡ As long as w ≠ 0, we can recover original point

$$p_{per} = M_{per} p = \begin{bmatrix} ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} ? \\ ? \\ ? \\ ? \end{bmatrix} = \begin{bmatrix} x/(z/d) \\ y/(z/d) \\ d \\ 1 \end{bmatrix}$$

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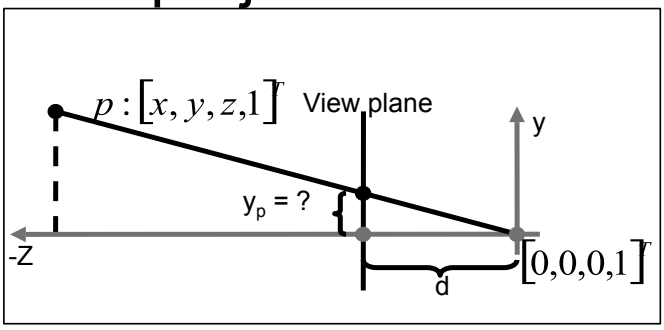

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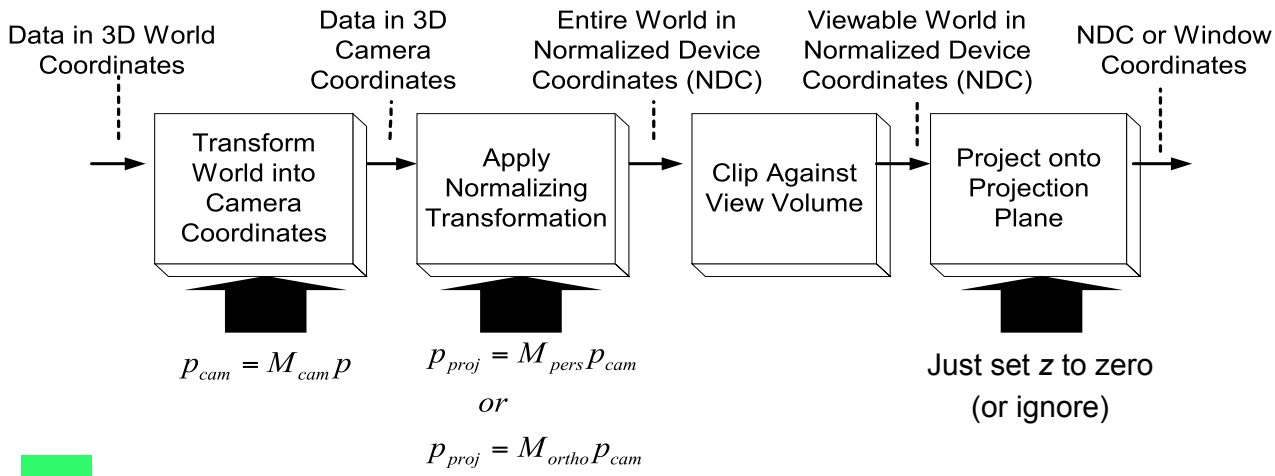
Perspective division

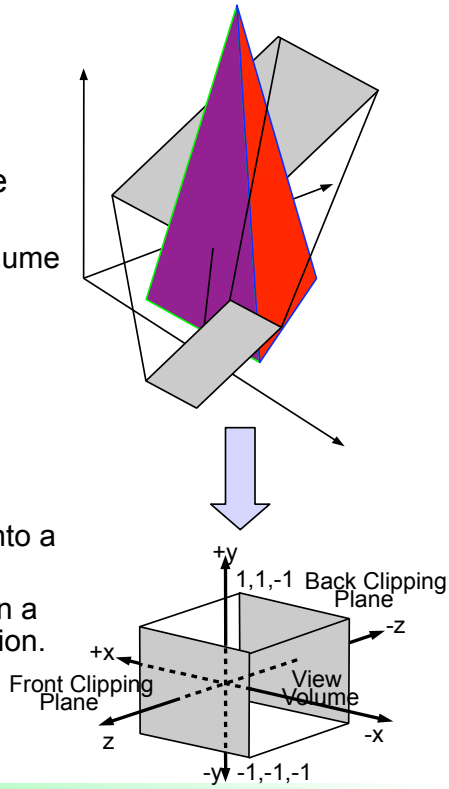
3D Viewing Transformation Projection Matrixes

- Input 3D World Coordinates
- Output 3D Normalized Device Coordinates
  - (a.k.a. Window Coordinates)



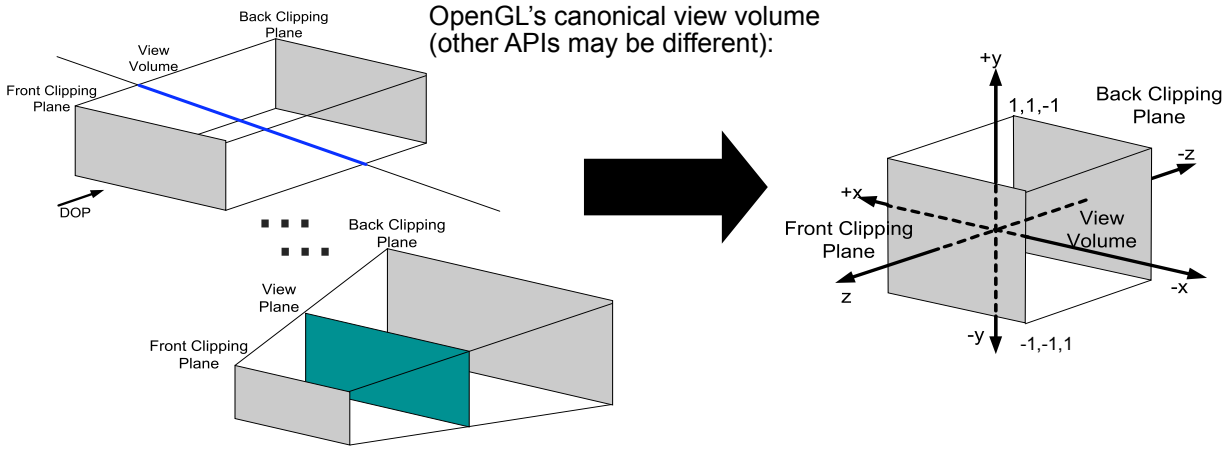
Projection normalization

- Several issues are not address with the simple projection matrices we have developed:
  - 3D Clipping Efficiency in a Frustum Viewing Volume
  - Hidden Surface Efficiency
- Solution: Use Projection Normalization
  - Get rid of perspective and other problem projections!
  - Everything is easier in an canonical, orthogonal world!
  - Distort world until viewing volume in world fits into a parallel canonical view volume.
  - Find a transformation that distorts the vertices in a way that we can use a simple canonical projection.



Idea: Canonical View Volume

- Define Viewing Volume via Canonical View Volumes
- Plus: Easier Clipping
- Minus: Another Transformation



Projection Normalization

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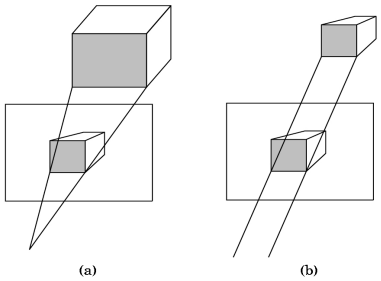
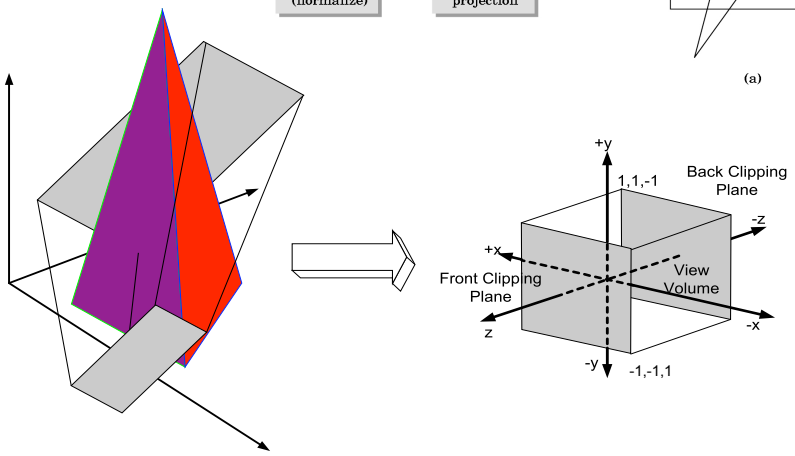
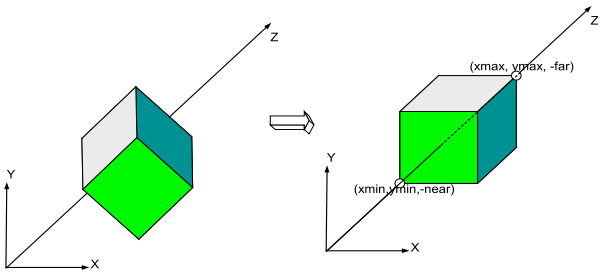
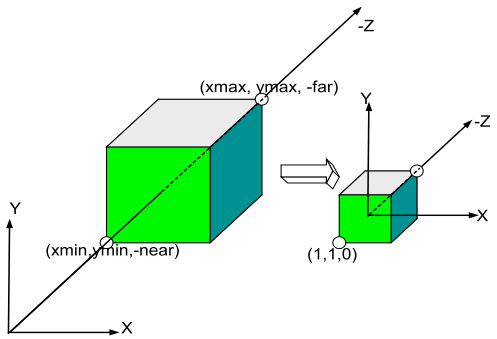




Camera Transformation and Projection Normalization for Orthogonal Views

- Camera Transformation in Orthogonal Views (M_{cam})
  1. Convert World to Camera Coordinates
     - Camera at origin, looking in the –z direction
     - Display plane center along the z axis
  2. Combinations of translate, scale, and rotate transformations
     - Can be accomplished through camera location specification
- Projection Normalization for Orthographic Views (M_{ortho})
  1. Translate along the z axis until the front clipping plane is at the origin
  2. Scale in all three dimensions until the viewing volume is in canonical form





Projection Normalization for Orthogonal Parallel Projections

Mapping a orthogonal view volume to a canonical view volume requires two affine transformations:

$$T_{\left(\frac{-(x_{\max}+x_{\min})}{2},\frac{-(y_{\max}+y_{\min})}{2},\frac{-(z_{\max}+z_{\min})}{2}\right)} \qquad S_{\left(\frac{2}{(x_{\max}-x_{\min})},\frac{2}{(y_{\max}-y_{\min})},\frac{2}{(z_{\max}-z_{\min})}\right)}$$

these can be concatenated to

$$M = ST = \begin{bmatrix} \frac{2}{x_{\max}-x_{\min}} & 0 & 0 & -\frac{x_{\max}+x_{\min}}{x_{\max}-x_{\min}} \\ 0 & \frac{2}{y_{\max}-y_{\min}} & 0 & -\frac{y_{\max}+y_{\min}}{y_{\max}-y_{\min}} \\ 0 & 0 & \frac{2}{z_{\max}-z_{\min}} & -\frac{z_{\max}+z_{\min}}{z_{\max}-z_{\min}} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The camera is pointing in the negative z direction. All projectors are from infinity towards the origin. Hence M can be written as:

$$M = \begin{bmatrix} \frac{2}{x_{\max}-x_{\min}} & 0 & 0 & -\frac{x_{\max}+x_{\min}}{x_{\max}-x_{\min}} \\ 0 & \frac{2}{y_{\max}-y_{\min}} & 0 & -\frac{y_{\max}+y_{\min}}{y_{\max}-y_{\min}} \\ 0 & 0 & \frac{-2}{far-near} & \frac{far+near}{far-near} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

OpenGL note:
OpenGL offers just an interface for the orthogonal case using
`glOrtho(xmin, xmax, ymax, ymin, near, far)`

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Finally, the resulting matrix has to be post multiplied by a simple orthogonal parallel projection

$$M = M_{orth}ST = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{2}{x_{\max}-x_{\min}} & 0 & 0 & -\frac{x_{\max}+x_{\min}}{x_{\max}-x_{\min}} \\ 0 & \frac{2}{y_{\max}-y_{\min}} & 0 & -\frac{y_{\max}+y_{\min}}{y_{\max}-y_{\min}} \\ 0 & 0 & \frac{-2}{far-near} & \frac{far+near}{far-near} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow M = \begin{bmatrix} \frac{2}{x_{\max}-x_{\min}} & 0 & 0 & -\frac{x_{\max}+x_{\min}}{x_{\max}-x_{\min}} \\ 0 & \frac{2}{y_{\max}-y_{\min}} & 0 & -\frac{y_{\max}+y_{\min}}{y_{\max}-y_{\min}} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$