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Modeling the Knowledge Base of Mathematics Learners: Situation-Specific and Situation-Nonspecific Knowledge

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Introduction

This chapter describes an approach to modeling the domain-specific knowledge of mathematics learners in a predicate-logic formalism suited for computer implementation. Two hypotheses are central to this approach. First, a person's cognitive behavior is a *knowledge-based process* that evolves from relatively simple component processes of an inferential nature. The complexity of a person's observed behavior in a domain depends on the knowledge base: how many facts and rules he or she has and how these facts and rules are organized. Second, a person's behavior in a task situation is generally not supported by the total body of his or her long-term knowledge. Rather, it is assumed that knowledge must be *activated* to be used in a given situation and that the accessibility of particular knowledge depends on contextual cues in the situation. The degree to which particular knowledge is contextually bound with respect to a set of specific situations is called *situation specificity*.

The activation of particular knowledge may depend on various kinds of contextual information. Significant context information is carried by the language involved in communicating a task situation. Language is a primary carrier of instructional transactions and is thus a dimension to be considered in modeling the knowledge of mathematics learners along with the dimension of operational knowledge as captured by the rule-based approach. This discussion focuses on the linguistic dimension.

The chapter begins with a brief discussion to motivate and exemplify the issue of situation specificity. A short introduction to computerized learner models follows. The major part of the chapter presents a logical programming approach to modeling student knowledge on the basis of a representation system implemented in the PROLOG (PROgramming in LOGic) language. Principles followed in modeling student knowledge are presented and discussed. Finally, an outline is given of how the approach

could lead to the development of a student modeling component in an intelligent tutoring system.

Situation Specificity and Inconsistent Student Behavior

It is a frequent observation in mathematics instruction that learners who master a task when it is posed in a standard setting may stumble when the "same" task is embedded in a new context; for example, an applied situation. A possible effect is that a learner gives different answers to a mathematical question posed in different contexts. In this sense the learner's behavior can be inconsistent across different situations involving the same sort of mathematics.

Empirical investigations have shown that learners' ability to apply knowledge of a subject domain cannot be considered independently of the context in which that knowledge was acquired (Seiler, 1973). When subjects have had to demonstrate their knowledge in settings that deviated from the situational context of instruction, they have not always been able to do so. While formal thinking structures arise from the individual's experience with specific problems in specific situations, they rarely reach an unrestricted, universal generality. Without further guidance an individual may not be able to apply a given rule in novel situations. Further, it is very probable that in one individual and with respect to one subject domain, different thinking structures can coexist that can become activated alternately, depending on the symbol system primarily triggered or cued by a situation (in particular *cf*. Seiler, 1973, p. 268).

From clinical research in the realm of rational number learning, Wachsmuth (1985a, 1985b) has presented examples that illustrate some of the points mentioned previously. One fifth-grade subject's behavior in comparing the size of several fractions gave evidence of having knowledge but being unable to use it optimally in an applied situation. The evidence indicated that the activation of knowledge was inhibited by the latency of, and a lack of mutual access to, relevant subdomains of fraction knowledge. It was hypothesized that the subject used one repertory of rules to make judgments about the equivalence or nonequivalence of fractions and another repertory of rules to determine the sequence (in magnitude) of nonequivalent fractions. When the second repertory was employed heavily in arranging a set of 12 fractions according to their order relationship as numbers (represented by different gray shadings), some fractions that had originally been recognized as being equivalent were treated as unequal. The activation versus nonactivation of the different sets of rules might explain why the subject exhibited inconsistent behavior with respect to stating the equivalence of certain fractions.

On the basis of another fifth grader's performance, two competitive

domains of fraction knowledge, specific to situations governed by certain language, were identified. Triggered by contextual cues, each domain could be activated independently of the other, but a connection across the different situations was lacking. When the interviewer contrasted contradictory answers given in the different contexts, the inconsistency in the subject's knowledge base caused a cognitive conflict to occur.

With respect to the psychology of learning, the issue of inconsistency is crucial. First, any serious attempt to improve instruction must recognize and deal with the fact that isolated, possibly incompatible, domains—"islands" of knowledge—can exist in the human mind and give rise to inconsistent behavior. Second, the discovery of inconsistencies can yield important hints about flaws in a learner's knowledge base and indicate where to invest remedial efforts. Identifying the conditions and laws of a student's inconsistent behavior prepares the grounds for remediation. Remediation so grounded will promote mature conceptions that are consistent and stable across a broad range of situations.

In summary it is the intent of any instruction to bring about knowledge that is widely applicable. That knowledge tends to remain situation specific seems to require particular instructional attention. Such restriction in a learner's developing cognitive structure might be overcome by intelligent guidance that diagnoses the learner's condition and evaluates appropriate · tutorial strategies. A central requirement for such an effort is that the cognitive structures of a learner be understood in terms of a framework that allows precise description of deficits.

Learner Models

Good teaching requires an understanding of the learner's thinking. A good teacher's instructional efforts are not restricted to preplanned behavior but can respond to a diagnosis and remediation of the learner's misconceptions. To make decisions about pedagogical interventions successfully, teachers must be able to put themselves in the learner's place, that is, make a model of the student's current thinking.

The construction and use of formal learner models is expected to pay off in improved instruction through better understanding of the organization of the learner's subject knowledge. Computerized learner models have become very important in attempts to apply artificial intelligence techniques for educational purposes in intelligent tutoring systems. In such a system a computer tutor diagnoses the student's errors and leads the student to an understanding of them. To do this the system uses the collected knowledge base of experienced teachers of the subject domain.

Three components comprise the general framework of an intelligent tutoring system (cf. Barr & Feigenbaum, 1982, pp. 229–235):

1. An "expert" component, which is charged with the task of generating problems and evaluating the correctness of the student's solutions
2. A student-model component, which is to represent the student's current understanding of the material to be taught
3. A tutoring component, incorporating knowledge about natural-language dialogues, teaching methods, and the subject area

The core of this approach is to compare, in a given problem situation, the student's actual response with an ideal interaction generated by the expert component (Figure 4.1). The difference will then be evaluated in order to make a decision about appropriate tutorial strategies.

In most instances so far, work has concentrated on the construction of single components of an intelligent tutoring system. Learner models are regarded as one of the most important components in the construction of intelligent tutoring systems but also have been found to be among the most difficult. A number of approaches have attempted to model individual students' understanding of the material to be taught; for example, by keeping catalogs of the student's response history or by setting "learned/not learned" flags in the rule base or in a subject-matter semantic net. Other approaches have modeled student knowledge as a deviation from expert knowledge. (For a more extensive review of this field cf. Barr & Feigenbaum, 1982, pp. 231–232). The issue of situation specificity is scarcely captured by such approaches, because they focus mainly on facts and rules, while the context-bound quality of such "particles of knowledge" is of concern here.

The notion of a learner model is concretized for the present purpose as follows: *A learner model is a system that makes concrete assumptions about a student's way of acting in specific situations.* The generality versus
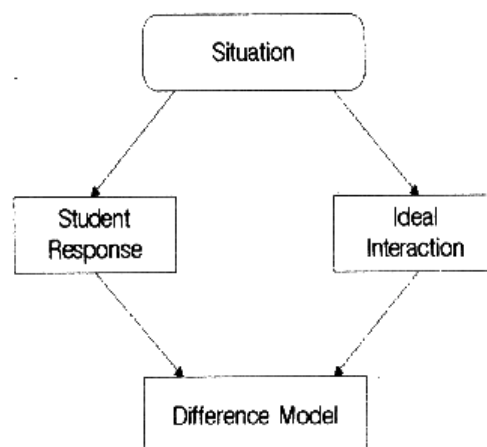

FIGURE 4.1. Difference model: Learner's suboptimal performance is explained as deviation from optimal performance.

specificity of a particular way of acting is then captured by the range (i.e., number and sort) of situations associated with it.

In order to design a computer-implementable representation for learner models, the first goal is to specify a *representation language* that can express pieces of student knowledge and model the use of such knowledge. A second goal of particular importance with respect to intelligent tutoring systems is the design of a component that generates and updates actualized hypotheses of individual learner knowledge.

As a means to describe and analyze the representation and use of domain-specific knowledge concisely, a formalized learner model, LAKOS, was developed at the University of Osnabrüeck. (LAKOS is an acronym standing for the German translation of logical analysis of cognitive organizational structures.) Its main intent is to derive hypotheses about the cognitive structures of individual learners. Such hypotheses, expressed in terms of the model, should provide "logical" explanations for learners' behavior even if the behavior appears irrational at first glance. A computerized version of the model, based on the technique of logical programming, has been developed. It models learner knowledge in terms of network structures as formulated by a human experimenter.

The LAKOS model emphasizes the following:

1. The linguistic competence of the learner, in the sense of what words are available to the learner, what meanings are associated with these, and in which contexts they are available and understood
2. The operational competence of the learner, in the sense of what abstract ways to act (rules) are available to the learner and in which situations they can be activated and used
3. The organization of the learner's knowledge as a basis for the flexibility of his or her performance
4. The disparity or connectedness of knowledge substructures
5. The generality or specificity with respect to the class of situations in which particular rules can be used

Although so far the construction of learners' knowledge structures must be accomplished by a human experimenter, these efforts can be an important precursor for conceptualizing intelligent tutoring systems. Whether our goal is to improve instructional strategies or to develop computerized teaching systems, the main objective of modeling a learner's knowledge structures remains the same: to obtain hypotheses about the learner's misconceptions and suboptimal behavior such that the teacher, or the system, can intervene in a corrective manner. In the same way that a good teacher should be able to understand the behavior of a learner, especially where it deviates from ideal behavior, intelligent tutoring systems should be able to diagnose origins of behaviors in terms of a learner model on which to base decisions about tutorial interventions.

Description of the LAKOS Model

The first implemented version of the model is the LAKOS1 system. It was conceptualized as a deductive question-answering system (Black, 1968) not restricted to a specific subject matter and was implemented in the PROLOG language (a Micro PROLOG version, MLOG, was used; Gust & Gust, 1984). The system can hold natural language dialogues of a restricted, standardized form with a user. The user proceeds by asking questions or probing behavior as if in a diagnostic interview. The computer takes the role of a person, some rudiments of whom are modeled in the machine, and answers questions or executes commands from the person's point of view. The system's responses are displayed on the terminal. They represent the actions or answer statements of the person as predicted by the model. If the user asks WHY the computer model gives a reason for its most recent answer.

The design of any such system requires both the specification of a representation scheme for bodies of facts and a method for deriving conclusions. In the LAKOS1 system, the representation scheme is a combination of formal logic and a network approach, and the reasoning method is deductive inference based on the resolution procedure (Robinson, 1965). As "world-dependent" components the knowledge base and the parser and generator need to be specified with respect to a specific application.

The reactions of the LAKOS1 system are generated as knowledge-based processes. The elements in the knowledge base are formulated as rules and facts. The rules are conditional statements, each consisting of one or more phrases, the antecedent(s), followed by an arrow, followed by another phrase, the consequent. Facts are included as rules without antecedents. In this approach there is no clear distinction between declarative and procedural knowledge. A rule has a declarative meaning as a descriptive statement about its constituents. In addition it has a procedural meaning by virtue of being executable by the interpreter. As is usual in the PROLOG language, rules are written in reverse, beginning with the consequent, interpreted as a goal that recurs on the antecedents as subgoals.

A prototypical instantiation of this model is the TERRI program, which was first presented in 1984 at the 5th International Congress on Mathematical Education in Adelaide (Carss, 1986; for more information cf. Wachsmuth, 1985a). Due to the economy of PROLOG, this program runs on an Apple II micro computer (with Z80 processor). It models responses from an uncertain pupil, not only to "straight" questions such as "Which is greater, $\frac{1}{4}$ or $\frac{1}{3}$?" but also to questions asking why the student gave a particular answer, for example, "$\frac{1}{4}$ is greater than $\frac{1}{3}$ [*sic*] because they have the same number on top and 4 is greater than 3." The student responses are not necessarily mathematically consistent but are modified in

the light of what wordings are used or what questions have already been asked by the user. Empirical clinical data from a long-term experimental teaching study carried out in the United States (the *Rational Number Project*[1]) served as a basis for the instantiation of the model.

As is seen in Figure 4.2, the LAKOS1 system consists of a dialogue interface, a knowledge base referred to as *long-term memory*, and three processes, PARSE, EVALUATE, and RESPONSE, which constitute components in the cognitive processing carried out by the system. Because of the limited subject matter for the TERRI program, it was possible to design a relatively simple natural language interface. (A parsing routine for arbitrary English sentences would be far more complex than the entire deductive system.) Further components of the system are a (semantic) *short-term memory* and a mechanism regulating the activation of knowledge coded in long-term memory, referred to as *focus*.

A working cycle of the system consists of three major steps:

1. PARSE transforms an input sentence into an expression in the representation language, activating a subset of the knowledge recorded in long-term memory.
2. EVALUATE searches the activated part of the knowledge base for relevant information and makes inferences to produce an answer internally.
3. RESPONSE generates a language answer and returns it to the terminal.

The results of the most recent inferences are kept in short-term memory for possible use in the evaluation of further queries. If the process fails at any step, an appropriate message is put out.

The knowledge in long-term memory is organized in the form of a *knowledge network*. The nodes in this network contain lexical language records and knowledge of a particular field of discourse in the form of rules that are interpreted as abstract ways to think and act. A single record in a node of the knowledge network is referred to by the term *knowledge element*. A knowledge element can be employed when it is marked active and when the data or part of the data of an input string match its structure.

The activation of knowledge is realized through the focus mechanism, which tags the network nodes that are currently accessible. The focus can shift along the links in the network during a dialogue in progress, causing a dynamic partitioning of the knowledge network into active and inactive knowledge. In this sense access structures are determined by the topology of the network. When a whole working cycle is completed, the focus re-
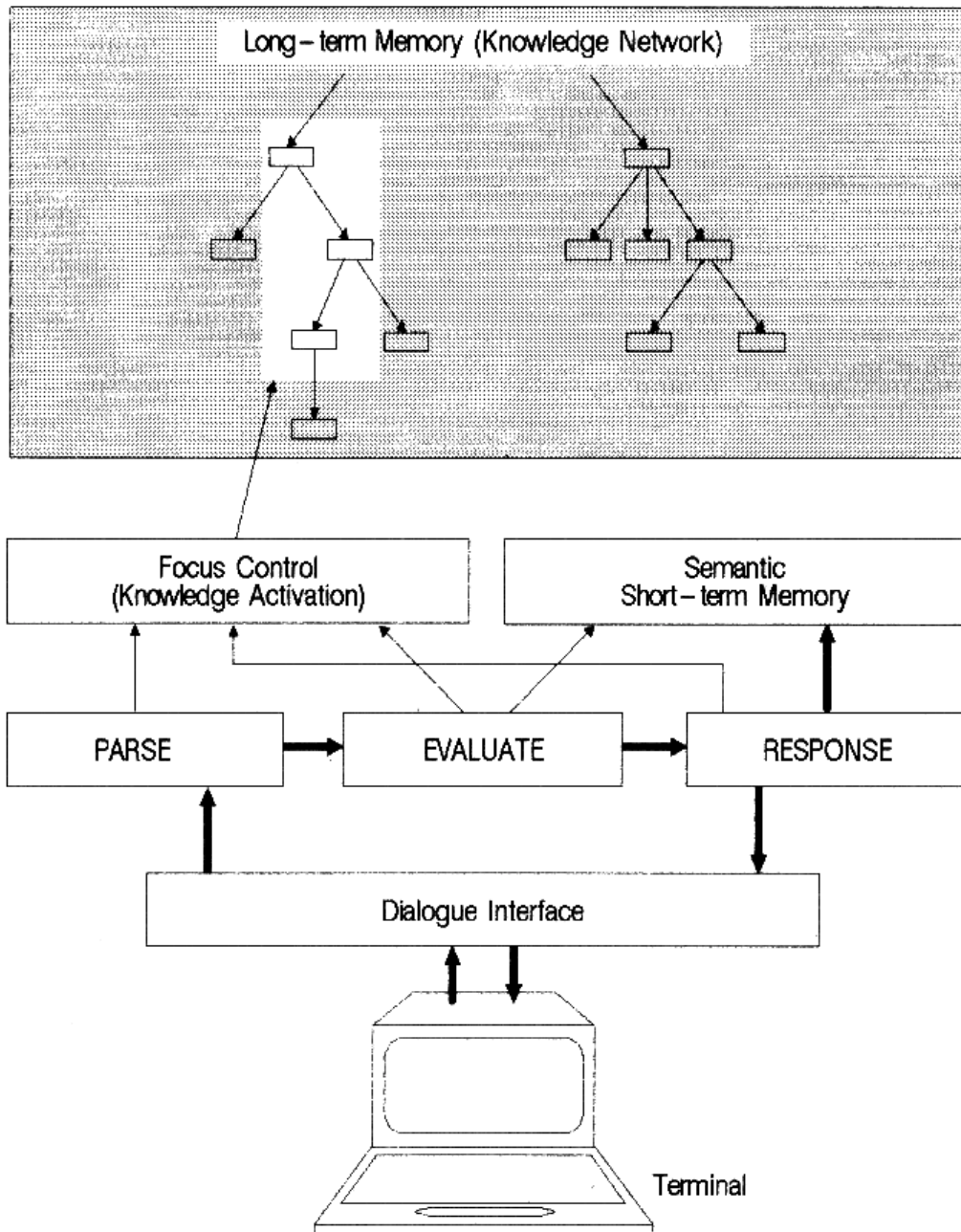

FIGURE 4.2. The LAKOS1 dialogue system. Light arrows denote access; heavy arrows denote information flow. Shaded region refers to inactive knowledge.

mains at the current node as starting focus for the next input which one could interpret as a "mind set".

The central idea of this modeling approach is that the potential actions an individual is able to perform are determined by his or her knowledge network. The explanatory power of the model thus lies in the fact that its

actions in the course of a simulated dialogue arise from the organizational structuring of its knowledge base.

Principles in the Modeling of Student Knowledge

As the basis for the model, it is assumed that individual structures of human memory—the "knowledge network"—are constituted by (1) sets of knowledge elements ("packets of knowledge") and (2) connections between these ("organizational network"). A knowledge packet is composed of a single node or a subnet consisting of several nodes (that is, a knowledge packet can be further structured). In the first instance only tree-structured networks were assumed; if there is a theoretical reason to do so, more general structures can also be represented.

The modeling of learner knowledge was exemplified in the realm of rational number learning with particular respect to size comparisons of fractions. Students' ability to make relative size judgments about fractions has been found to be an indicator for their development of a quantitative understanding of rational numbers (Behr, Wachsmuth, Post, & Lesh, 1984).

Based on the general model discussed previously, the hypothetical knowledge structures of individual learners concerning size comparisons of fractions were described in a tree-structured network. Modular pieces of learner knowledge were derived from subject answers given in clinical interviews and were captured in rules that were stored in indexed memory nodes. We present the general guidelines we followed to represent the operational competence of particular students:

1. Partition the subject domain into subclasses that require specific ways to act.
2. Select test items to assess the student's performance with respect to these subclasses.
3. Formulate rules based on the student's explanations.
4. Specify an appropriate node index to integrate a rule in the knowledge network.

For example, the following major subclasses of the particular subject domain, size comparisons of fractions, were distinguished (cf. Behr et al., 1984):

SN: Comparison of same-numerator fractions, for example, $\frac{3}{4}$ and $\frac{3}{5}$

SD: Comparison of same-denominator fractions, $\frac{6}{15}$ and $\frac{12}{15}$

GE: Comparison of general fractions, $\frac{5}{6}$ and $\frac{8}{9}$ or $\frac{4}{6}$ and $\frac{6}{9}$

Some words of explanation just for the first two subclasses: While children's early performance is frequently found to be dominated by whole-number schemas (e.g., "one third is less than one fourth because three is less than four"), they will eventually need to separate their thinking from the whole-number schemas and acquire a rule that puts fractions with the same numerators in the reversed order relation with respect to their denominators. In contrast the order relation of same-denominator fractions is consistent with the order relation of the whole numbers in the numerators. But here it is sometimes observed that at one stage the new rule that puts same-numerator fractions in correct order is overgeneralized and used to order same-denominator fractions in reversed fashion (e.g., "three fifths is less than two fifths"). At a later stage this kind of overgeneralization may be prevented by way of further discrimination of task characteristics (same numbers in the numerators vs. in the denominators).

TABLE 4.1. Formulation of rules based on subject responses to items in fraction subclasses early in teaching experiment.

Sample answers in subclass SN

$\frac{3}{6}$ and $\frac{3}{8}$

   "Three eighths [is less]." *Explain.* "It takes more to cover."

$\frac{3}{6}$ and $\frac{3}{8}$

   "One is less. Three eighths." *Explain.* "It would take more to cover the unit."

Source

   Items B1-3.1 and B1-3.6, Bert (age 10;0)

Verbal description

   "The second one of two fractions with equal numerator is less, if the first denominator is less than the second one."

Rule

   $(> (*X*Y) (*X*V)) < = (\text{LESS}*Y*V)$

Sample answers in subclass SD[a]

$\frac{3}{5}$ and $\frac{5}{6}$

   "One's less. Five sixths." *Explain.* "It would take more to cover."

$\frac{3}{6}$ and $\frac{5}{6}$

   "Three sixths." *Explain.* "Oh, no, five sixths is less" (shakes his head in his hands).
   Explain. "It takes more to cover the unit."

Source

   Items B1-4.1 and B1-4.6, Bert (age 10;0)

Verbal description

   "The second one of two fractions with equal denominator is less, if the first numerator is less than the second one."

Rule

   $(> (*X*Y) (*U*Y)) < = (\text{LESS}*X*U)$

SD, same denominators; SN, same numerators.
[a] Incorrect ordering of SD fractions due to overgeneralization of reversed order relation.

Such stages in the gradual development of a learner's ability to master tasks in the item classes mentioned can be captured in the model by different knowledge networks that model different levels of the learner's competence with respect to the taxonomy of the tasks. Tables 4.1 and 4.2 show basic examples of how student answers were used to obtain rules.[2]

Rules of thumb were derived from the experimental use of the model with the objective of reproducing protocols from interview sessions with subjects. Such "rules of thumb" were used to specify the node indexes. For example:

- Rules that are older with respect to the student's learning history are put in "higher" nodes, whereas rules acquired more recently are put in subordinate nodes.

TABLE 4.2. Formulation of rules based on subject responses to items in fraction subclasses later in teaching experiment.

Sample answer in subclass SN[a]

$\frac{5}{12}$ and $\frac{5}{9}$

"One's less. Five twelfths." *Explain.* "Well, the pieces, the twelfths are smaller, so ... but the ... that means they're smaller, the larger number on the bottom or top is smaller. ... If the top number is the same, then the larger number on the bottom means that's smaller."

Source
  Items II-2.2, Bert (age 10;2)
Verbal description
  "The first one of two fractions is less if the numerators are equal and the second denominator is less than the first one."
Rule
  $(< (*X*Y) (*U*V)) < = (EQ*U*X) (LESS*V*Y)$

Sample answer in subclass SD[a]

$\frac{6}{7}$ and $\frac{9}{7}$

"One's less. Six sevenths." *Explain.* "There are not as many pieces covered or shaded."
Source
  Item II-3.1, Bert (age 10;2)
Verbal description
  "The first one of two fractions is less if the denominators are equal and the first numerator is less than the second one."
Rule
  $(< (*X*Y) (*U*V)) < = (EQ*Y*V) (LESS*X*U)$

SD, same denominators; SN, same numerators.
[a] Differentiation between size of pieces (reflected by denominator) and number of pieces (reflected by numerator) prevents overgeneralization.

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[2] Terms like "cover", "pieces", etc. refer to imagined physical representations of fractions as were used in the instruction.

- Rules that are observed in a common context are given the same node index.
- Rule nodes that are observed to be accessible from one another are linked to lie on a path.
- Rule nodes that are observed to be disparate are given separate access paths from the superordinate node.

As explained in the introduction, the contextual dependency of a learner's operational competence is (partly) reflected in the way that the student understands certain words. The representation of learner knowledge uses the following guidelines to incorporate this linguistic aspect:

- Identify "significant" words with respect to the subject domain, that is, words that possess a specific meaning as distinguished from everyday language or words that are observed to trigger certain behavior. (Significant words serve to activate parts of the knowledge base.)
- "Rules of thumb":
  The more generally available a significant word, the "higher" is the node index for integrating the word in the knowledge network; in the extreme, a word that is observed to be available in all contexts with the same meaning is put in the highest node.

  The more specific a significant word, the "lower" is the node index for the word; in the extreme, a word available in only one context ("key word") is put in a leaf node (cf. Table 4.3).

  Two words, each of which is observed to be available in only one context at the same degree of generality/specificity, obtain the same node index.

Whereas words that are understood across different contexts give access to a larger part of the knowledge base, key words limit the rules available for inferences to the rules on the path to the leaf node that holds the key word. With respect to the network representation, there is no principal distinction between language knowledge and operational knowledge other than declaring particular entries to be of type TALK or type RULE. As "elements" of a learner's knowledge, both types of entries may be put in the same node. Thus, particular words may be associated with particular ways to act.

Toward a Learner Module in an Intelligent Tutoring System

The LAKOS model was developed primarily with a psychological intent, namely, to obtain a better notion of the way in which the organizational structuring of the "knowledge base" of a learner gives rise to particular kinds of behavior. So far, a human experimenter formulates the descrip-

TABLE 4.3. Identification of key words based on situational dependence of subject response to items within subclass SD.

Question was: (fraction1) and (fraction2), are they *equal* or is *one less*?

$\frac{9}{24}$ and $\frac{7}{24}$

"They're equal. Same size pieces and it takes the same amount to cover and the same size pieces. [gestures] "They're equal." *OK, what about the 9?* "That'd be nine pieces and seven pieces.... They're equal." *What exactly do you mean by "they are equal"?* "They have the same size pieces, so you know."

$\frac{6}{15}$ and $\frac{12}{15}$

"They're equal. Because they have the same denominator."

Source
Items VII-2.4 and VII-3.2, Terri (age 11;6)
Key phrase
EQUAL OR ONE LESS

Directions were to arrange fractions in *order*

$\frac{6}{15}$ and $\frac{12}{15}$

(Puts $\frac{6}{15}$ left of $\frac{12}{15}$) *Explain* "Because 6 comes before 12 so I thought that's the way you do it."

Source
Item VII-3.1, Terri (age 11;6)
Key word
ORDER

SD, same denominators.

tions that specify the knowledge base of an individual student on the basis of assessments of the student's performance and explanations. To do this the experimenter makes judgments about how to capture particles of the student's knowledge in rules and about how to integrate them in a knowledge network. In a sense the experimenter acts as an expert in the formalization of student knowledge, using heuristics, rules of thumb, and so on as previously described.

The following discussion explores how the approach presented could lead into the construction of a learner-model module to be incorporated in an intelligent tutoring system. Although these ideas are preliminary and none has yet been implemented in such a system, they may help to clarify possible directions for further work.

The question to be attacked in the context of intelligent tutoring systems is how to go about having an automated learner-modeling component generate hypotheses about a learner's domain-specific knowledge in the course of instructional sessions. Two things are necessary. First, the design of a representation system for learner knowledge and, second, the design of processes to generate and update assumptions about the user of the tutoring system during teaching dialogues and diagnostic assessments.

Although the modeling approach presented in the preceding sections seems to cover some of the requirements for a representation system, the second topic has yet to be dealt with. In principle it requires that the experimenter's expertise in representing learner knowledge be made explicit enough to be captured in rules that can be executed by a computer.

A hard approach, which certainly would involve a lot of effort, might be to let the system conduct diagnostic dialogues with the user on the basis of which the rules are inferred by the system. Technically, it does not seem totally absurd to parse students' explanations to obtain strings in the semantic representation language. These could then serve to be generalized into rules. But if this idea were technically realized, it would probably be at high cost, at least on the basis of the technology currently available for the processing of natural language. Furthermore, bounds would probably be reached when students could not sufficiently explain their actions.

A way that seems much more feasible at the present stage is the following. An empirical screening in the particular field of subject matter will make known many strategies that students use. Some strategies—both correct and incorrect—will be common to many learners. Such data are available from the Rational Number Project (e.g., Behr et al., 1984), and probably from work in a number of other areas. A catalog of possible rules formally describing such strategies can then be incorporated into the system, grouped by subclasses within which they are ordered by increasing sophistication (e.g., in terms of the number of subgoals in a rule). The following steps could yield a description of a learner's current knowledge in the domain:

- Match the student's performance on selected test items with rules in the relevant subclass.
- For each item choose the first (i.e., simplest) rule that produces the same response as the student.
- Choose an adequate node index (according to the subclass), and integrate the rule in the knowledge network.

Although a rule selected in this way may not completely mimic the student's actual thinking, it at least captures the student's behavior in the sense of an "axiomatic characterization."

A major problem to be dealt with in this approach occurs when a student uses idiosyncratic rules with outcomes that are not produced by any of the rules in the catalog. Another problem appears when a student exhibits inconsistent behavior even within a subclass of items without variation of contextual conditions. For example, a student may know that $\frac{1}{2}$ equals $\frac{2}{4}$ but will order less familiar equivalent fractions according to some whole number relationships of numerators and denominators, like $\frac{4}{6}$ less than $\frac{6}{9}$.

Although this case could be dealt with by creating subnodes that allow further discrimination of item characteristics, great problems would occur when a student responded inconsistently to different presentations of the same test item with no situational variations observable.

The next question would be how to change the rule base when changes are observed in consecutive diagnostic assessments carried out by the system. So far, in the psychological approach to the implementation of a reproductive simulation model, no rule previously employed is ever taken off the network. (This allows modeling processes of "backsliding" to seemingly eradicated behaviors.) Rather, if a "new" rule is diagnosed that produces different behavior in situations that were already included in the model, a constraint is imposed on the "old" rule, which intercepts its employment when inadequate by making finer discriminations of situational characteristics. Although the justification for this way of modeling is explicitly psychological, it would probably be sensible for an intelligent tutoring system to keep track of students' "old" rules in order to recognize fallbacks.

This article raised the issue of situation specificity to make an argument that tutoring must not be approached too naively. If the aim of a tutoring system is to bring about progress in a learner's ability to utilize knowledge in a broad range of situations, then the following two general objectives for such a system should be taken into account:

- To help the learner master a set of rules that can support successful performance in the subject matter in question
- To enable the learner to use these rules in a sufficiently varied set of situations to ensure that the learner's rules will be evoked in a variety of contexts

These objectives are derived from the following pragmatic assumption: *Only when learners exhibit consistent success with a variety of applied situations involving a subject matter can they be assumed to have developed a sufficiently general understanding of the subject matter to predict success in an even broader class of situations.*

Consequently, the question arises, how shall we represent situational characteristics of learner knowledge in the student module of an intelligent tutoring system? Earlier we suggested that situational competence is characterized by the learner's command of certain language. A possible way to model the situation specificity of a learner's rules, then, might be the following: We need to link a node holding rules of operations relevant to certain situations as a superordinate to nodes holding linguistic units that characterize those situations. Then the relevant rules would become active by activation of any subordinate node. In case only a single situation node can trigger activation of a rule node, that knowledge would have to be regarded as situation specific. The more specific situations are represented in nodes subordinate to a rule node, the broader the range of situations

in which that knowledge can be activated. If all in a predetermined set of situational descriptions selected for instructional tutoring are found to be linked to the corresponding rule node, then that knowledge would be termed situation nonspecific (with respect to the objectives of the tutoring system).

In the LAKOS model as prototypically specified, levels of different specificity of a student's rules can be distinguished with respect to situations typified by certain language. We have presented some ideas about how this approach could be developed into a student-model module of an intelligent tutoring system. These ideas are still far from full realization and exploitation. Probably the hardest problem to be dealt with is the diversity of reasons that cause learners to make errors (see the illuminating discussion of this issue by Davis, 1982). At the present stage one may be modestly optimistic that progress in the modeling of student knowledge will make some sort of "intelligent" tutoring possible upon further advancement of current developments.

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