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C^2 Continuous Gait-Pattern Generation for Biped Robots

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Abstract

In this paper, we propose a new method to generate C^2 continuous gait motion for biped robots. The method is based on the enhanced inverted pendulum mode, which can easily handle angular momentum around the center of gravity. Using our method, it is possible to plan motion paths for biped robots without discontinuity in the acceleration even during switching from single support phase to double support phase, and vice versa.

1 Introduction

Biped locomotion of humanoid robots is one of the most exciting topics these days. Many researchers have proposed humanoid robots which realize biped gait [4, 2, 1, 3]. One way to generate gait motion is to convert a dynamically infeasible motion to a feasible one [7, 5]. This requires good quality motion in advance, such as motion captured data. Another way is to generate gait motion based on the Inverted Pendulum Mode (IPM) [4, 3]. Using the IPM, we can simplify complex models of humanoid robots which have too many degrees of freedom to be controlled directly. However, the IPM doesn't consider any angular momentum, since it assumes that the the center of gravity (COG) is a mass point and that the ground force vector always passes through the COG of the system. Because of this, it is difficult to generate various patterns of gait motion, particularly "human-like gait".

In this paper, we propose a new method to plan gait motion for biped robots in real-time. The proposed method, named the Angular Momentum inducing Inverted Pendulm Mode (AMPM), is based on the IPM, which allows real-time motion generation for biped robots, but two new concepts are added. This AMPM enables to generate motion (1) which considers angular momentum around the COG, and (2) is C^2 continuous at the moment of switching from single support phase to double support phase, and vice versa.

The method follows a bottom-up approach: First, trajectory of the center of gravity and rotational momentum of the whole body during the motion are calculated. Then, the motions of body segments that satisfy the predefined trajectories are calculated using inverse kinematics. By changing the value of the parameters that determine the angular momentum around the COG, it is possible to generate gait motion that could not be created by the IPM without destabilizing the system due to discontinuous acceleration.

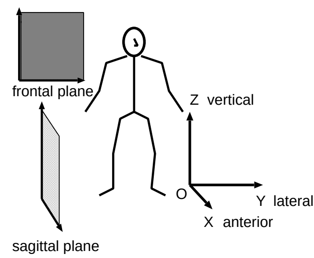


Figure 1: *Posture of the human body model in the world coordinate system.*

1.1 Coordinates used in this study

We assume in this paper that the human body stands in the world coordinate system as shown in Figure 1. The x-axis corresponds to the anterior axis, y-axis to the lateral axis, and z-axis to the vertical axis. The frontal plane is defined by the lateral and vertical axis, and the sagittal plane is defined by the anterior and vertical axis.

1.2 Inverted Pendulum Model

The IPM is often used in robotics to calculate trajectories of the COG of humanoid robots. This model assumes the ground reaction force to pass through the COG, as shown in Figure 2. Using the IPM, the differential equation to describe the COG motion in the world coordinate system can be written as

$$\ddot{x}_g = \frac{g}{H}\,x, \quad \ddot{y}_g = \frac{g}{H}\,y$$

where H is the height of the COG which is considered to be constant, g is the gravity constant ($g = 9.81$),

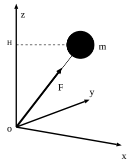


Figure 2: *IPM. The ground force always penetrates the COG.*

x_g is the position along the anterior axis, and y_g is the position along the lateral axis. The great advantage of using this model is that the motion of the COG in the x-z plane and in the y-z plane can be calculated independently.

By setting boundary conditions, the trajectory of the COG can be written as the following equations:

$$\begin{aligned} x &= x_0 \cosh \frac{t}{T_c} + \dot{x}_0 T_c \sinh \frac{t}{T_c} \\ y &= y_0 \cosh \frac{t}{T_c} + \dot{y}_0 T_c \sinh \frac{t}{T_c} \end{aligned}$$

where $T_c = \sqrt{H/g}$, (x_0, y_0) and $(\dot{x}_0, \dot{y}_0)$ are the coordinates and velocity of the COG in the x-y plane at time $t = 0$. Although the ground force is applied to the whole sole of the foot, it is known that such a force can be summarized to a specific point. This point is known as the zero moment point (ZMP) [6] in the field of robotics. Therefore, the ZMP can be considered as the center of the ground reaction force. The ZMP of the humanoid robot is considered as the origin of this coordinate system.

2 Enhanced Inverted Pendulum Model

A new model of the COG is proposed here, which enhances the IPM in two ways: (1) the ZMP is allowed to move over the ground, (2) the ground force vector does not have to be parallel to the vector between the ZMP and the COG, as far as its horizontal element is linearly dependent on the COG position.

As a result, rotational moment is allowed to be generated by the ground force. Their relationship is depicted in Figure 3. The position of the COG is (x, H), the position of the ZMP is $(ax + b, 0)$, and the normal vector of the ground force is parallel to vector $(cx + d, H)$. The relationship between the acceleration of the COG and its position becomes:

$$F_x : F_y = \ddot{x} : (\ddot{z} + g) = (cx + d) : H.$$

As the height of the COG is $z = H$, we can write

$$\ddot{x} = \frac{g}{H}(cx + d). \quad (1)$$

The solution for this differential equation can be written as

$$x = C_1 e^{-\frac{t}{T_e}} + C_2 e^{\frac{t}{T_e}} - \frac{d}{c}$$

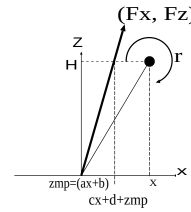


Figure 3: *The AMPM. The ZMP is allowed to move over the ground, and its position must be linearly dependent to that of the COG. The horizontal component of the ground force vector is allowed to change, by an amount which must be linearly dependent on the COG.*

where $T_e = \sqrt{H/(cg)}$, and C_1, C_2 are constant values. As initial parameter values are set at $x = x_0$ and $\dot{x} = v_0$ at $t = 0$, the constant values C_1, C_2 are

$$C_1 = \frac{x_0 + \frac{d}{c} - v_0 T_e}{2}, \quad C_2 = \frac{x_0 + \frac{d}{c} + v_0 T_e}{2}.$$

Then, the ground force vector can be written as

$$\begin{aligned} F_x &= m\ddot{x} = \frac{m}{{T_e}^2}\left(C_1 e^{-\frac{t}{T_e}} + C_2 e^{\frac{t}{T_e}}\right) \\ F_z &= mg \end{aligned}$$

where m is the mass of the system. The rotational moment r around the y-axis can be calculated as

$$\begin{aligned} r = &-\frac{mH}{{T_e}^2}\left(C_1 e^{-\frac{t}{T_e}} + C_2 e^{\frac{t}{T_e}}\right) \\ &-mg\left((a-1)\left(C_1 e^{-\frac{t}{T_e}} + C_2 e^{\frac{t}{T_e}} + \frac{d}{c}\right) + b\right) \end{aligned}$$

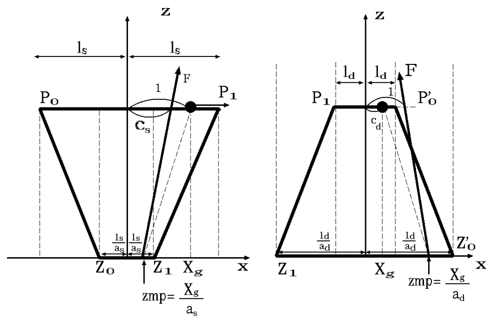
and the angular momentum ω_{t_1,t_2} generated by the rotational momentum between times $t = t_1, t_2$ can be obtained as

$$\begin{aligned} \omega_{t_1,t_2} = m \Bigg[&\left(-C_1 e^{-\frac{t}{T_e}} + C_2 e^{\frac{t}{T_e}}\right)\left(-\frac{H}{T_e} - gT_e(a-1)\right) \\ &-g\left((a-1)\frac{d}{c} + b\right) t \Bigg]_{t_1}^{t_2} + \omega_1 \end{aligned}$$

where ω_1 is the angular momentum at $t = t_1$. In the following sections, we show how AMPM can be used to generate COG trajectories in the frontal and lateral plane.

2.1 Application of the AMPM for motion in the sagittal plane

Gait motion consists of two phases, which are the single support phase and the double support phase. The model is supported by one leg during the single support phase and it is supported by two legs during the double support phase. The relationship between



(a) (b)

Figure 4: *The relationship of the COG, ZMP, and ground force during (a) the single support phase and (b) the double support phase. The positions of the COG and ZMP are defined here by* (X_g, H) *and* $(\frac{X_g}{a_s}, 0)$ *during the single support phase, and* (X_g, H) *and* $(\frac{X_g}{a_d}, 0)$ *in the double support phase. The x coordinate value of the ZMP is considered linear to that of the COG. During the single support phase, the COG travels from* P_0 *to* P_1*, and the ZMP travels from* Z_0 *to* Z_1*, as well. During the double support phase, the COG travels from* P_1 *to* P_0'*, which is the starting point of another single support phase, and the ZMP travels from* Z_0 *to* Z_0'*. The position of* P_0, P_1, Z_0, Z_1 *in the single support coordinate system are defined here by* $(-l_s, H), (l_s, H), (-\frac{l_s}{a_s}, 0)$*, and* $(\frac{l_s}{a_s}, 0)$*, where* $2l_s$ *is the distance the COG travels during the single support phase, and* $2l_d$ *is the distance the COG travels during the double support phase. The positions of* P_1, P_0', Z_1, Z_0' *in the double support coordinate system are defined here by* $(-l_d, H), (l_d, H), (-\frac{l_d}{a_d}, 0)$*, and* $(\frac{l_d}{a_d}, 0)$*.*

COG, ZMP and ground force during these phases is assumed here as shown in Figure 4, and the relationship between the acceleration of the COG and its position during the single support phase and the double support phase can be written relatively as

$$\ddot{x}_g : (\ddot{z}_g + g) = x_g\left(c_s - \frac{1}{a_s}\right) : H, \qquad (2)$$

$$\ddot{x}_g : (\ddot{z}_g + g) = x_g\left(c_d - \frac{1}{a_d}\right) : H, \qquad (3)$$

where c_s, a_s, c_d and a_d are parameters which are described in Figure 4. Solving these as in the previous subsection, the trajectory of the COG during the two phases is obtained as

$$x_g = C_{s1}\cosh\frac{t}{T_s} + C_{s2}\sinh\frac{t}{T_s} \quad \text{(single)}$$

$$x_g = C_{d1}\cos\frac{t}{T_d} + C_{d2}\sin\frac{t}{T_d} \quad \text{(double)},$$

where C_* are constants and

$$T_s = \sqrt{\frac{H}{(c_s - 1/a_s)g}}, \quad T_d = \sqrt{\frac{H}{(1/a_d - c_d)g}}.$$

Therefore, using AMPM, the motion of the COG in the sagittal plane can be expressed by hyperbolic functions during the single support phase, and by trigonometric functions during the double support phase. This is because $c_s - 1/a_s > 0$ and $c_d - 1/a_d < 0$. As we assume the acceleration of the COG must be continuous when switching from single support phase to double support phase, and *vice versa*, the ground force vector at the switching moment in the single coordinate system (Figure 4(a)) and the double support coordinate system (Figure 4(b)) must be the same. By setting $x_g = l_s$ in Equation 2 and $x_g = -l_d$ in Equation 3, we obtain the following condition:

$$\frac{l_s}{c_s} + \frac{l_d}{c_d} = l_s + l_d. \qquad (4)$$

Therefore, if the constant values c_s, c_d, l_s, l_d are chosen in a manner that satisfies Equation 4, the acceleration becomes continuous.

As shown in Figure 4, the motions of the COG during the single support phase and the double support phase are symmetric. The angular momentum generated in the initial half of the phase is compensated by that in the latter half. Therefore, it is not necessary to tune parameters to avoid divergence of the angular momentum.

2.2 Application of AMPM for motion in the frontal plane

The coordinate systems used here are shown in figure 5. These systems are the same as those defined in Kajita et al [3]. The distance between the feet when they are both on the ground is $2s + 2\beta$. The COG travels 2β during the double support phase. After switching to single support phase, it travels along until it stops and returns back the same path. The analytical models of the single support phase and the double support phase can be explained by AMPM. The way they are modeled is explained in the following.

The relationship between ZMP, COG and ground force during the single support phase is

$$\ddot{y}_g : (\ddot{z}_g + g) = c_y y_g : H. \qquad (5)$$

where c_y is a constant value that can be set by the user (Figure 5(b)). Using the terminal condition, the motion of the COG can be finally written as:

$$y_g = s\cosh\frac{t}{T_{ls}} - v_{ex}\sinh\frac{t}{T_{ls}}. \qquad (6)$$

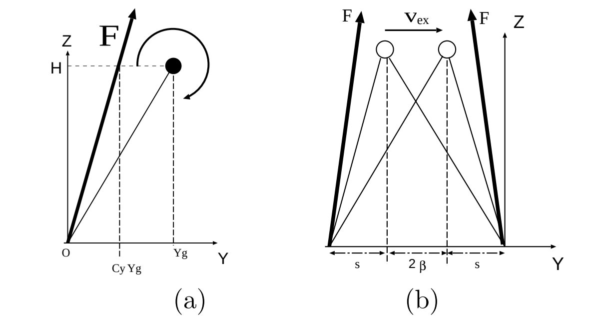


Figure 5: *The coordinate systems used in the frontal plane for (a) single support phase, and (b) double support phase. The origin is set to the position of the ZMP in single support phase, and to the center in double support phase.*

where v_{ex} is the velocity when the single support phase starts, and $T_{ls} = \sqrt{H/(c_y\, g)}$. Since the duration of the single support phase T is determined by the motion in the sagittal plane ($T = t_2 - t_0$), v_{ex} can be calculated by setting $t = T, y_g = s$ into Equation 6. As a result, v_{ex} can be calculated as

$$v_{ex} = \frac{-s + s\cosh\frac{T}{T_{ls}}}{\sinh\frac{T}{T_{ls}}}$$

The y-component of the ground force can be written as

$$\begin{aligned} F_y &= \frac{m}{T_{ls}^2}\left(y_0 \cosh\frac{t}{T_{ls}} - v_{ex}\sinh\frac{t}{T_{ls}}\right) \\ F_z &= -mg \end{aligned}$$

As the trajectories of the ground force, COG, and ZMP are known, the rotational moment around the anterior axis can be calculated as:

$$r_x = y_g F_z - H F_y.$$

The double support phase can be modeled as follows. Since the motion in the frontal plane is symmetric with respect to time, we can assume the ZMP and COG satisfy the following relationship:

$$z_y = \frac{\beta}{\beta + s}\, y_g.$$

Since rotational momentum is generated in the frontal plane, and since rotational momentum decreases as the COG approaches the origin of the coordinate system, the motion of the COG can be approximated by the following function:

$$\ddot{y}_g : g = c_3(y_g - z_y) : H, \tag{7}$$

where c_3 is a constant. Using the boundary conditions of the single support phase, c_3 can be calculated as

$$c_3 = \frac{\beta + (1 - a_{\mathrm{cog}y})s}{\beta} < 0.$$

Then, the trajectory of y_g becomes

$$y_g = C_1 \cos\frac{t - t_2}{T_c} + C_2 \sin\frac{t - t_2}{T_c} - \frac{b}{T_c^2}.$$

where C_1, C_2 are arbitrary constant values. Using the terminal conditions, the final form becomes:

$$y_g = (\beta + s)\cos\frac{t - t_2}{T_c} + v_e x \sin\frac{t - t_2}{T_c}.$$

The y and z components of the ground force can be written as

$$\begin{aligned} F_y &= -\frac{m}{T_c^2}\left((\beta + s)\cos\frac{t - t_2}{T_c} + v_{ex}\sin\frac{t - t_2}{T_c}\right) \\ F_z &= -mg \end{aligned}$$

Rotational moment around the anterior axis can be calculated by using Equation 7, same as in the sagittal plane. Because of the symmetry of the motion with respect to time, we do not have to tune any parameters to compensate for the angular momentum.

2.3 Calculating the joint angles using inverse kinematics

As we have already defined the trajectories of the COG and the angular momentum, the next step is to calculate kinematic parameters that satisfy these trajectories. Inverse kinematics is used for this purpose. A human body model with 40 degrees of freedom, as shown in Figure 6, was used. At first, positions and rotational trajectories of the feet, which are defined here as (p_l, θ_l) and (p_r, θ_r), are calculated using the foot step data specified in advance. Four key-frames of the support foot are specified as shown in Figure 7. The data includes the posture of the foot at initial contact, initial full contact, heel rise, and toe off. The x- component of the velocity of the foot of the swing leg when it is lifted from the ground is calculated by

$$v^0_{\mathrm{swing}} = \frac{l_s}{T_{\mathrm{swing}}}.$$

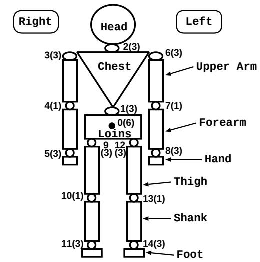


Figure 6: *The human body model used in this study*

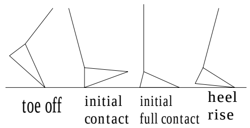


Figure 7: *The key-frames of the foot rotation*

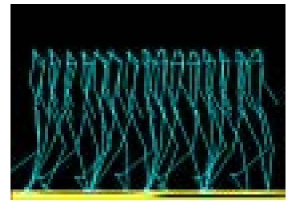


Figure 8: *Side view of the gait motion trajectory*

The final velocity when it lands on the ground is set to zero. The trajectory of the swung foot is calculated by interpolating the key-frames with a cubic spline curve.

Trajectories of generalized coordinates of the human body model are defined here as $\boldsymbol{q}(t) = (q_1(t), q_2(t), ..., q_{\mathrm{dof}}(t))^T$ where dof is the number of degrees of freedom of the human body model. Generalized coordinates $\boldsymbol{q}(t)$ include the position and rotation of the root of the body in the 3D world coordinate system.

The relationship between velocity of the COG and velocity of the generalized coordinates can be written as follows:

$$\dot{\boldsymbol{x}}_g = J_{\mathrm{cog}}\,\dot{\boldsymbol{q}},$$

where J_{cog} is the Jacobian matrix that consists of the partial differentials of the COG by the generalized coordinates:

$$J_{\mathrm{cog}} = \frac{\partial \boldsymbol{x}_g}{\partial \boldsymbol{q}}.$$

Then, the acceleration of the COG can be obtained as follows:

$$\ddot{\boldsymbol{x}}_g = J_{\mathrm{cog}}\,\ddot{\boldsymbol{q}} + \dot{J}_{\mathrm{cog}}\,\dot{\boldsymbol{q}}. \quad (8)$$

Angular momentum $\boldsymbol{r}$ and first derivative of the generalized coordinates have a linear correlation:

$$\boldsymbol{r} = R\dot{\boldsymbol{q}}.$$

Then, the derivative of the angular momentum can be calculated as follows:

$$\dot{\boldsymbol{r}} = R\ddot{\boldsymbol{q}} + \dot{R}\dot{\boldsymbol{q}}. \quad (9)$$

Acceleration of the feet can be expressed as functions of $\ddot{\boldsymbol{q}}$ as well:

$$\begin{pmatrix} \ddot{\boldsymbol{p}}_l \\ \ddot{\boldsymbol{p}}_r \\ \ddot{\boldsymbol{\theta}}_l \\ \ddot{\boldsymbol{\theta}}_r \end{pmatrix} = J_f\ddot{\boldsymbol{q}} + \dot{J}_f\dot{\boldsymbol{q}} \quad (10)$$

Combining Equation 8, 9, and 10, linear constraints that must be satisfied can be written in the following form:

$$\boldsymbol{\lambda} = J_{\mathrm{all}}\,\ddot{\boldsymbol{q}} + \dot{J}_{\mathrm{all}}\,\dot{\boldsymbol{q}}. \quad (11)$$

where $\boldsymbol{\lambda} = (\ddot{\boldsymbol{x}}_g, \dot{\boldsymbol{r}}, \ddot{\boldsymbol{p}}_l, \ddot{\boldsymbol{\theta}}_l, \ddot{\boldsymbol{p}}_r, \ddot{\boldsymbol{\theta}}_r)^T$, and $J_{\mathrm{all}} = (J_{\mathrm{cog}}, R, J_f)^T$. Calculating $\ddot{\boldsymbol{q}}$ that satisfies Equation 11 can be considered an inverse kinematics problem.

Since the goal is to calculate a stable gait motion, $\ddot{\boldsymbol{q}}$ that minimize the following quadratic form is calculated here:

$$(\ddot{\hat{\boldsymbol{q}}} - k(\hat{\boldsymbol{q}} - \hat{\boldsymbol{q}}_0) + d\dot{\hat{\boldsymbol{q}}})(\ddot{\hat{\boldsymbol{q}}} - k(\hat{\boldsymbol{q}} - \hat{\boldsymbol{q}}_0) + d\dot{\hat{\boldsymbol{q}}})^T. \quad (12)$$

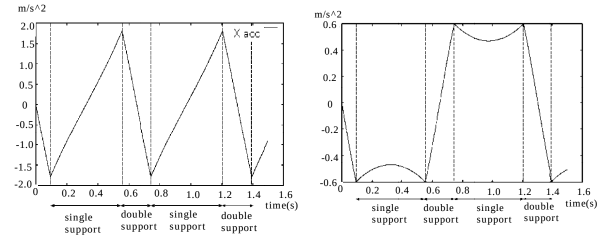
where $\ddot{\hat{\boldsymbol{q}}}$ is the subset of $\ddot{\boldsymbol{q}}$ which determines an upright posture of the body. Those parameters include rotation of the loins and joint angles of the chest. $\hat{\boldsymbol{q}}_0$ is the target posture to keep the body upright, which is a zero vector here, and k, d are the elastic and the viscosity constants respectively.

$\ddot{\boldsymbol{q}}$ that minimize Equation 12 and satisfy Equation 11 were calculated through quadratic programming. Using the calculated acceleration, the values of the generalized coordinates and their velocity were updated step by step, and finally, the whole trajectory was obtained.

3 Experiments

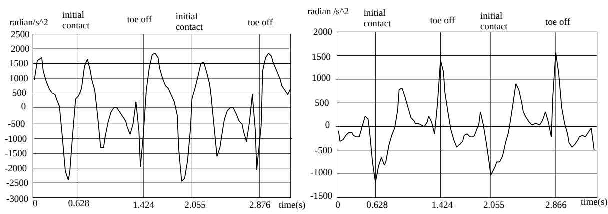
In this section, several gait motions which have various angular momentum are generated. First, the trajectory of the COG is determined, then the trajectories of the ZMP and ground force are calculated from the trajectory of the COG, and then the posture of the whole body is calculated using inverse kinematics. Parameters in equations are set manually and angular momentum during motion can be controlled by setting these appropriately.

The first demo of normal gait is shown in Figure 8. The step length is set to 0.6m, the initial velocity in the single support phase to 1.0m/s. The acceleration of the COG during this gait along the anterior axis and lateral axis are shown Figure 9. As can be seen, the acceleration values are continuous even when switching from single support phase to double support phase. The angular acceleration of the flexion/extension of (a) the hip joint, and (b) the knee joint is shown in Figure 3. As can be seen, the acceleration of these joints is also continuous. Next,



(a) (b)

Figure 9: *The acceleration of the COG along (a) the anterior and (b) the lateral axis*



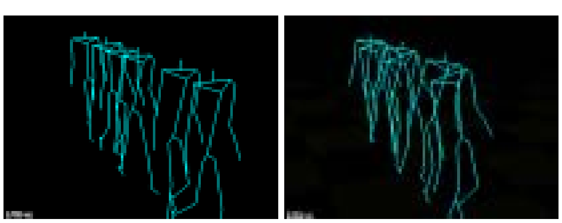
(a) (b)

Figure 10: *The angular acceleration of the flexion/extension of (a) the hip joint, and (b) the knee joint*

the angular momentum around the frontal axis during the motion is increased by setting the value of c_y in Equation 5 from 0.1 to 0.7. The model swings its upper body in the frontal plane according to the magnitude of the applied moment. The final demo is a motion in which the angular momentum around the y-axis is enlarged (Figure 3). When a large moment is applied around the y-axis, the upper body swings largely in the sagittal plane and the model easily loses its balance. In order to prevent this, parameters are set to make each step large in this demo.

4 Summary and Future Work

In this paper, we proposed a new approach to generate gait motion. The algorithm, the Enhanced Inverse Pendulum Model, is based on the Inverse Pendulum Model. It allows to include moment around



(a) (b)

Figure 11: *An example of increasing the rotational momentum around the anterior axis c_y from (a) 0.1 to (b) 0.7.*

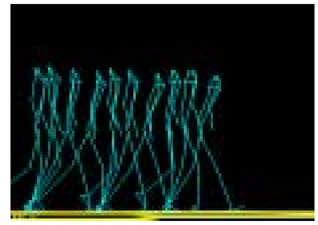


Figure 12: *The trajectory of the humanoid model when the angular momentum around the lateral axis is large.*

the COG and guarantees C^2 continuity during the whole motion. Two elements of motion, in the sagittal plane and the frontal plane, can be calculated independently in this model, and a variety of motions can be generated by changing parameter values. After calculating the trajectories of the COG and the ZMP using AMPM, the whole body posture is determined by inverse kinematics.

In future work, we plan to use this gait motion generator to approximate various kinds of gait motion, and characterize the features of those motions with the parameters defined in this study. This will be particularly useful for generating various kinds of gait motion by selecting a number of parameters from a database.

References

[1] Yasutaka Fujimoto and Atsuo Kawamura. Robust biped walking with active interation control between robot and environment. *Proceedings of IEEE AMC96*, 1996.

[2] K Hirai, M Hirose, Y Haikawa, and T. Takenaka. The development of honda humanoid robot. *Proceedings of IEEE International Conference on Robotics and Automation*, pages 1321–1326, 1998.

[3] Shuuji Kajita, Osamu Matsumoto, and Muneharu Saigo. Real-time 3d walking pattern generation for a biped robot with telscopic legs. *Proceedings of the 2001 IEEE International Conference on Robotics and Automation*, 2001.

[4] Shuuji Kajita, Tomio Yamaura, and Akira Kobayashi. Dynamic walking control of a biped robot along a potential energy conserving orbit. *IEEE Transactions on Robotics and Automation*, 8(4), 1992.

[5] Seyoon Tak, Oh young Song, and Hyeong-Seok Ko. Motion balance filtering. *Computer Graphics Forum*, 19(3), 2000.

[6] M. Vukobratović, B. Borovac, D.Surla, and D.Stokić. *Biped Locomotion*. Springer-Verlag, 1990.

[7] Katsu Yamane and Yoshihiko Nakamura. Dynamics filter - concept and implementation of on-line motion generator for human figures. *Proceedings of IEEE International Conference on Robotics and Automation*, pages 688–694, 2000.