# Spezielle Themen der Künstlichen Intelligenz

7. Termin:

Uncertainty, Degrees of Belief and Probabilities



# Uncertainty

Intelligent agents need to cope with uncertainties in their...

### knowledge

- incomplete: partially observable, noisy sensors, non-deterministic environments
- incorrect: world and beliefs may differ

### reasoning & action selection

- reasoning rules may be not correct, or not fully applicable
- conclusions might be less or more uncertain than their antecedents
- actions may have unpredictable effects (bounded or undounded indeterminacy)
- deducing all consequences may be too complex, need to do pruning based on approximation and heuristics

# Recap‘: handling uncertainty in planning

## Sensorless planning (conformant planning)

- find plan that achieves goal in all possible circumstances, often not possible

## Conditional planning (contingency planning)

- construct conditional plan with different branches for possible contingencies
- gets intractable fast, need to skip contingencies in plan

## Execution monitoring & replanning

- while constructing and executing a plan, judge whether plan requires revision

## Continuous planning

- planner persists over time: adapt plan to changed circumstances, reformulate goals if necessary



# Acting under uncertainty

Let action $A_t$ = *leave for airport t minutes before flight*

<u>*Question*</u>: Will $A_t$ get me there on time?

What are the problems for a purely logical agent?

# Acting under uncertainty

A purely logical approach either

- risks falsehood: "$A_{25}$ will get me there on time" = *true*
- leads to conclusions too weak and unreliable for decision making

*Example*:

- $A_{90}$ will get me there on time *if* there's no accident on the bridge *and* it doesn't rain *and* my tires remain intact *and* .....
  - plan success not inferrable (qualification problem)

Logical agent would be unable to act rationally

- *Instead*: rational decision depends on both relative importance of goals and likelihood that they will be achieved to the necessary degree



# Acting under uncertainty

*Idea in a nutshell*

Use probabilistic assertions (not propositions) to summarize effects of

- laziness: failure to enumerate exceptions, qualifications, etc.
- ignorance: lack of relevant facts, initial conditions, etc.

Subjective probability relates facts to the own state of knowledge

- degree of belief, e.g., Pr($A_{25}$ | no reported accidents) = 0.06
- *not* a degree of truth, i.e. no assertions about the world, only about belief

Probabilities of assertions change with new evidence

- posterior or conditional probabilites:
  Pr($A_{25}$ | no reported accidents, 5 a.m.) = 0.15

# Acting under uncertainty

*Idea in a nutshell*

Suppose the agent believes the following:

- $Pr(A_{25}$ gets me there on time | …) = 0.04
- $Pr(A_{90}$ gets me there on time | …) = 0.70
- $Pr(A_{120}$ gets me there on time | …) = 0.95
- $Pr(A_{1440}$ gets me there on time | …) = 0.999

Which action to choose depends on *preferences* for possible outcomes (risks, costs, rewards, etc.) represented using utility theory

- decision theory = probability theory + utility theory

**Principle of maximum expected utility (MEU)**

An agent is rational **iff** it chooses the action that yields the highest expected utility, averaged over all possible outcomes of the action



# Acting under uncertainty

*Idea in a nutshell*

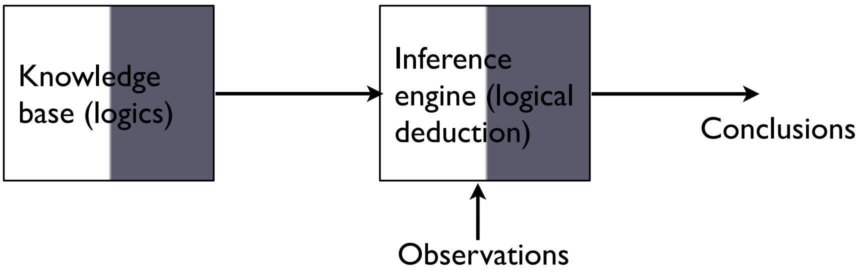
```
function DT-AGENT(percept) returns eine Aktion
   static: belief_state, probabilistischer Glauben über den aktuellen Zustand
           der Welt
           action, die Aktion des Agenten

   aktualisiere belief_state basierend auf action und percept
   berechne Ergebniswahrscheinlichkeiten für Aktionen
      abhängig von Aktionsbeschreibungen und aktuellem belief_state
   wähle action mit dem höchsten erwarteten Nutzen
    für gegebene Wahrscheinlichkeiten der Ergebnisse und Nutzeninformation
   return action
```

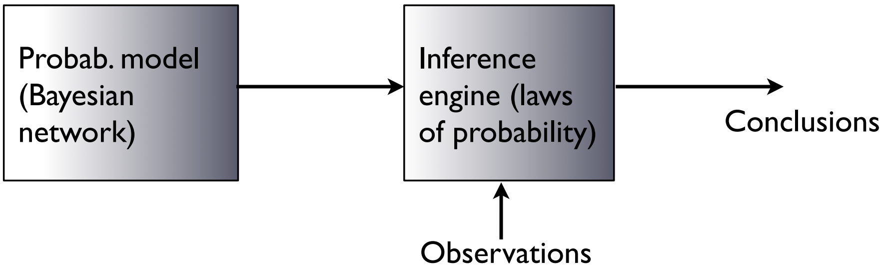
Decision-theoretic Agent

# From propositions to degree of beliefs

## Classical knowledge-based (or model-based) reasoning




## Probabilistic reasoning






# Propositional logics

World = state of affairs in which each propositional variable is known

- variable assignment with values

$$\omega$$

Models = worlds that satisfy a sentence

- every sentence represents a *set of worlds* = (atomic) event

$$Mods(\alpha) = \{\omega : \omega \models \alpha\}$$

| *World* | **Earthquake** | **Burglary** | **Alarm** |
|---|---|---|---|
| *w1* | *true* | *true* | *true* |
| *w2* | *true* | *true* | *false* |
| *w3* | *true* | *false* | *true* |
| *w4* | *true* | *false* | *false* |
| *w5* | *false* | *true* | *true* |
| *w6* | *false* | *true* | *false* |
| *w7* | *false* | *false* | *true* |
| *w8* | *false* | *false* | *false* |

$$Mods(\alpha \wedge \beta) = Mods(\alpha) \cap Mods(\beta)$$
$$Mods(\alpha \vee \beta) = Mods(\alpha) \cup Mods(\beta)$$
$$Mods(\neg\alpha) = \overline{Mods(\alpha)}$$

# Propositional logics

Important properties of sentences

- consistent / satisfiable $Mods(\alpha) \neq \{\}$
- valid $Mods(\alpha) \neq \Omega$ $\models \alpha$

Important relationships of sentences

- equivalent $Mods(\alpha) = Mods(\beta)$
- mutually exclusive $Mods(\alpha) \cap Mods(\beta) = \{\}$
- exhaustive $Mods(\alpha) \cup Mods(\beta) = \Omega$
- implies $\alpha \models \beta$ $Mods(\alpha) \subseteq Mods(\beta)$



# Monotonicity of logical reasoning

| *World* | **Earthquake** | **Burglary** | **Alarm** |
| --- | --- | --- | --- |
| *w1* | *true* | *true* | *true* |
| *w2* | *true* | *true* | *false* |
| *w3* | *true* | *false* | *true* |
| *w4* | *true* | *false* | *false* |
| *w5* | *false* | *true* | *true* |
| *w6* | *false* | *true* | *false* |
| *w7* | *false* | *false* | *true* |
| *w8* | *false* | *false* | *false* |

$\alpha : (Earthquake \vee Buglary) \Rightarrow Alarm$

$Mods(\alpha) = \{\omega_1, \omega_3, \omega_5, \omega_7, \omega_8\}$

+

$\beta : Earthquake \Rightarrow Burglary$

$$Mods(\alpha \wedge \beta) = Mods(\alpha) \cap Mods(\beta) = \{\omega_1, \omega_5, \omega_7, \omega_8\}$$

Monotonicity

learning new information can *only rule out* worlds:

- if *a* implies *c*, then (*a and b*) will imply *c* as well

Especially problematic in light of qualification problem! (why?)

# Modeling degrees of belief as probabilities

Degree of belief or probability of a *world* $Pr(\omega)$

- in fuzzy logic, interpreted as *possibility* (*not* the view adopted here)

Degree of belief or probability of a *sentence*

$$Pr(\alpha) := \sum_{\omega \models \alpha} Pr(\omega)$$

State of belief or joint probability distribution

$$\sum_{\omega_i} Pr(\omega_i) = 1$$

| *World* | **Earthquake** | **Burglary** | **Alarm** | *Pr(.)* |
|---|---|---|---|---|
| *w1* | *true* | *true* | *true* | *.0190* |
| *w2* | *true* | *true* | *false* | *.0010* |
| *w3* | *true* | *false* | *true* | *.0560* |
| *w4* | *true* | *false* | *false* | *.0240* |
| *w5* | *false* | *true* | *true* | *.1620* |
| *w6* | *false* | *true* | *false* | *.0180* |
| *w7* | *false* | *false* | *true* | *.0072* |
| *w8* | *false* | *false* | *false* | *.7128* |

$$Pr(Earthquake) = .1$$
$$Pr(Burglary) = .2$$
$$Pr(Alarm) = .2442$$



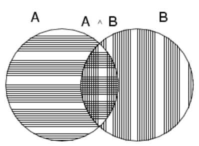
# Properties of beliefs

Properties of (degrees of) beliefs

- bound $0 \leq Pr(\alpha) \leq 1 \quad \forall \alpha$
- baseline for inconsistent sentences $Pr(\alpha) = 0 \; \forall \alpha \; inconsistent$
- baseline for valid sentences $Pr(\alpha) = 1 \quad \forall \alpha \; valid$

Junctions of beliefs

- disjunction $Pr(\alpha \vee \beta) = Pr(\alpha) + Pr(\beta) - Pr(\alpha \wedge \beta)$
- conjunction $Pr(\alpha \wedge \beta) = 0 \; if \; \alpha, \beta \; mutually \; exclusive$




$$Pr(Earthquake \wedge Burglary) = Pr(\omega_1) + Pr(\omega_2) = .02$$
$$Pr(Earthquake \vee Burglary) = .1 + .2 - .02 = .28$$

# Uncertainty and entropy

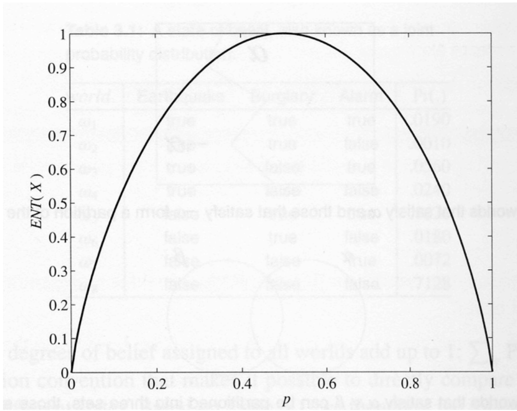
Entropy = quantifies uncertainty about a certain variable

$$ENT(X) := -\sum_x Pr(x) log_2 Pr(x)$$

$$(0\ log 0 := 0)$$

| World | Earthquake | Burglary | Alarm | Pr(.) |
|---|---|---|---|---|
| w1 | true | true | true | .0190 |
| w2 | true | true | false | .0010 |
| w3 | true | false | true | .0560 |
| w4 | true | false | false | .0240 |
| w5 | false | true | true | .1620 |
| w6 | false | true | false | .0180 |
| w7 | false | false | true | .0072 |
| w8 | false | false | false | .7128 |

| | Earthquake | Burglary | Alarm |
|---|---|---|---|
| true | .1 | .2 | .2442 |
| false | .9 | .8 | .7558 |
| ENT(.) | .469 | .722 | .802 |






# Updating beliefs

Evidence = a piece of information known to hold $\beta$

➔ requires to update state of belief with certain certain properties $Pr(.) \rightarrow Pr(.|\beta)$

- accommodate evidence $Pr(\beta|\beta) = 1$ $Pr(\omega|\beta) = 0 \quad for\ all\ \omega \models \neg\beta$
- normalized $\sum_{\omega \models \beta} Pr(\omega|\beta) = 1$
- retain impossible worlds $Pr(\omega) = 0 \rightarrow Pr(\omega|\beta) = 0$
- retain relative beliefs in possible worlds $\frac{Pr(\omega)}{Pr(\omega')} = \frac{Pr(\omega|\beta)}{Pr(\omega'|\beta)}$

$$\forall \omega, \omega' \models \beta, Pr(\omega) > 0, Pr(\omega') > 0$$

# Updating beliefs

➔ update old state of beliefs through conditioning on evidence $\beta$

$$Pr(\omega|\beta) := \begin{cases} 0 & \omega \models \neg\beta \\ \frac{Pr(\omega)}{Pr(\beta)} & \omega \models \beta \end{cases}$$

new beliefs = old beliefs, normalized with old belief in new evidence

| Earthquake | Burglary | Alarm | *Pr(.)* |
|---|---|---|---|
| *true* | *true* | *true* | *.0190* |
| *true* | *true* | *false* | *.0010* |
| *true* | *false* | *true* | *.0560* |
| *true* | *false* | *false* | *.0240* |
| *false* | *true* | *true* | *.1620* |
| *false* | *true* | *false* | *.0180* |
| *false* | *false* | *true* | *.0072* |
| *false* | *false* | *false* | *.7128* |

Alarm=*true* →

| Earthquake | Burglary | Alarm | *Pr(.\|Alarm)* |
|---|---|---|---|
| *true* | *true* | *true* | *.0190/.2442* |
| *true* | *true* | *false* | *0* |
| *true* | *false* | *true* | *.0560 /.2442* |
| *true* | *false* | *false* | *0* |
| *false* | *true* | *true* | *.1620 /.2442* |
| *false* | *true* | *false* | *0* |
| *false* | *false* | *true* | *.0072 /.2442* |
| *false* | *false* | *false* | *0* |

$$Pr(Burglary) = .2 \rightarrow Pr(Burglary|Alarm) = .741$$



# Updating beliefs

More efficient: direct update of a sentence from new evidence through Bayesian conditioning

$$Pr(\alpha|\beta) = \frac{Pr(\alpha \wedge \beta)}{Pr(\beta)}$$

follows from the following commitments

- worlds that contradict evidence have zero prob
- worlds that have zero prob continue to have zero prob
- worlds that are consistent with evidence and have positive prob will maintain their relative beliefs

<u>*Note*</u>: Bayesian conditioning is nothing else than application of the basic product rule

$$Pr(\alpha \wedge \beta) = Pr(\alpha|\beta) \cdot Pr(\beta)$$

# Updating beliefs

*Example*: State of belief from above

| | Pr(Earthquake) | Pr(Burglary) | Pr(Alarm) |
|---|---|---|---|
| *true* | *.1* | *.2* | *.2442* |

Conditioning on first evidence: *Alarm=true*

| | Pr(E\|Alarm) | Pr(B\|Alarm) | Pr(A\|Alarm) |
|---|---|---|---|
| *true* | *.307* | *.741* | *1* |

Conditioning on second evidence: *Earthquake=true*

| | Pr(E\|A∧E) | Pr(B\|A∧E) | Pr(A\|A∧E) |
|---|---|---|---|
| *true* | *1* | *.253* | *1* |

Belief dynamics under incoming evidence is a consequence of the initial state of beliefs one has !!



# Independence

A given state of beliefs finds an event independent of another event **iff**

$$Pr(\alpha|\beta) = Pr(\alpha) \text{ or } Pr(\beta) = 0$$

Equivalent definition (using product rule): $Pr(\alpha \wedge \beta) = Pr(\alpha) \cdot Pr(\beta)$

*Examples & properties:*

- in the initial state of beliefs defined above, it is
  - *Pr(Earthquake)*=.1 and *Pr(Earthquake* | *Burglary)*=.1
  - *Pr(Burglary)*=.2 and *Pr(Burglary* | *Earthquake)*=.2

  ➔ *Earthquake* and *Burglary* are independent, knowing one doesn't change belief in the other
- independence (property of beliefs) is always symmetrical
- ...but different from mutual exclusiveness (property of events)

# Conditional Independence

## Independence is a dynamic notion!

- *Earthquake* and *Burglary* are dependent when having evidence *Alarm*
  - *Pr(Burglary | Alarm)=.741* and *Pr(Burglary | Alarm∧Earthquake)=.253*

  ➔ *Earthquake* changes the belief in *Burglary* in presence of *Alarm*
- can also be the other way around (dep. →*evidence*→ indep.)

*Definition:*

state of belief Pr finds $\alpha$ conditionally independent of $\beta$ given event $\gamma$ **iff**

$$Pr(\alpha|\beta \wedge \gamma) = Pr(\alpha|\gamma) \text{ or } Pr(\beta \wedge \gamma) = 0$$

- conditional independence is always symmetric

$$Pr(\alpha \wedge \beta|\gamma) = Pr(\alpha|\gamma)Pr(\beta|\gamma) \text{ or } Pr(\gamma) = 0$$



# Conditional Independence

*Example*:

Given two noisy, unreliable sensors

| Temp | sensor1 | sensor2 | Pr(.) |
|---|---|---|---|
| normal | normal | normal | .576 |
| normal | normal | extreme | .144 |
| normal | extreme | normal | .064 |
| normal | extreme | extreme | .016 |
| extreme | normal | normal | .008 |
| extreme | normal | extreme | .032 |
| extreme | extreme | normal | .032 |
| extreme | extreme | extreme | .128 |

Initial beliefs

- *Pr(Temp=normal)=.80*
- *Pr(Sensor1=normal)=.76*
- *Pr(Sensor2=normal)=.68*

After checking sensor1 and finding its reading is *normal*

- *Pr(Sensor2=normal | Sensor1=normal) ~ .768* ➔ initially dependent

But after observing that temperatur is *normal* ....

- *Pr(Sensor2=normal | Temp=normal) = .80*
- *Pr(Sensor2=normel | Temp=normal, Sensor1=normal) = .80* ➔ become independent