Universität Bremen

One Focus or Many?

Modeling Attentional Distributions During Spatial Term Use

Master Thesis

for attainment of the academic degree of

Master of Science

in Computer Science

presented by

Thomas Kluth

matriculation number:

277 503 9

First Supervisor

Dr. Holger Schultheis

Second Supervisor

Prof. Dr. Carsten Lutz

Bremen, March 13, 2014

Für meine Eltern.

Contents

1 Introduction

> *"In der Tat bestehen die meisten Irrtümer nur darin, daß wir die Wörter nicht richtig auf die Dinge anwenden."*
> — Baruch de Spinoza, *Ethik, zweites Buch, Lehrsatz 47*

If one asks a passerby how many dimensions make up our world, the most likely answer will be: three dimensions, namely width, height and depth. All of these dimensions are space-based: x, y, z. However, it might be that you will get a different answer[1]. The second most likely answer, I assume, will be: four dimensions, thus adding time to space and by doing so obtaining space-time.

Space is an important part of our world. For example, we have to extract informations from space ("Where are my shoes?") or plan actions in space ("Please, can you pass me the salt shaker?") on a daily basis. We are able to do most of these things automatically. But how do we do this? What are the mechanisms guiding us through space, day by day, almost effortlessly?

A subtopic of this rather complex question is the question of how spatial language maps the visual world. Consider the utterance "Please, can you pass me my keys? They are *to the left of* the computer monitor." This seems to be a simple task. But what to do, if the keys are not at the location you assumed from the instruction? One might imagine what the speaker has in mind while searching all possible locations on the desk. Where is the *left* the speaker referred to? Is it relative to my viewpoint? Or, relative to the orientation of the monitor? Finally, the keys are found: "Here are your keys, I found them *to the front right of* the monitor."

Keeping this example in mind, it is reasonable to think that referring to space by using language is a possible source of confusion. The subject of the present work are the mechanisms underlying the production and apprehension of spatial terms, such as *to the left of* or *above*.

Concretely, a seminal cognitive model (the AVS model by Regier and Carlson (2001)) was implemented. Due to the fact that functional relationships between objects affect our use of spatial language, a functional extension to the AVS model (fAVS) was proposed by Carlson, Regier, Lopez, and Corrigan (2006). However, this extension implicitly assumes the ability of humans to split their visual spatial attention. Since it is controversially debated if humans are able to do so, I developed and implemented three alternative extensions without this assumption. Thus, this work[2] examines the role of visual spatial

[1] You should not conduct this test on a university campus, since people from different subjects would come up with a lot of more possible answers (e.g. a physician: "Infinite!", a philosopher: "What world?", a mathematician: "Please, define your problem more specifically.", ...).

[2] Parts of this work were submitted to the Spatial Cognition conference 2014 (Kluth & Schultheis, submitted).

attention in spatial term use.

The main object of this work is a comparison of the AVS model, the fAVS model and the three alternative functional extensions proposed in the present work. The AVS model is found to closely fit human rating data based on abstract geometric shapes, i.e., without functional effects (Regier & Carlson, 2001). However, the fAVS model is not as well assessed as the AVS model. The present work provides quantitative model simulations with pertinent empirical data for all five models.

Moreover, by evaluating these five models, I quantitatively compare the assumption of a multimodal attentional distribution with the assumption of an unimodal attentional distribution. Thus, the results of this work also contribute to the debate whether visual spatial attention can be divided or not.

1.1 Structure of this work

The structure of the present work is as follows: First, I will give an overview about the research in the domain of spatial prepositions (Chapter 2). Then, I will move on to introduce attention (Chapter 3), especially visual attention, since the AVS model, discussed in Chapter 4, explains spatial term use with the allocation of visual spatial attention. In Chapter 4, I will also introduce the functional extension to the AVS model (fAVS) as well as propose three alternative functional extensions.

Chapter 5 describes the method and data used to compare these five models. In Chapter 6, the results of the comparison are shown. This chapter also contains a discussion about the implications of the results. Finally, I will conclude the present work in Chapter 7.

2 Spatial Prepositions

There are approximately 80 to 100 spatial prepositions in the English language (Landau & Jackendoff, 1993, cited in Coventry and Garrod (2004)). According to (Coventry & Garrod, 2004, p. 7-8) these spatial prepositions can be divided in *locative/relational* and *directional* prepositions, see Figure 2.1.

> Locative or relational prepositions describe the location of one object in relation to another (...), whereas directional prepositions describe a change of position (...) or direction in which an object is located (...). (Coventry & Garrod, 2004, p. 8)

Relational prepositions can be further distinguished into *projective/dimensional* and *topological* prepositions. “Topological terms include prepositions such as *in*, *on*, and *near*, which usually refer to (static) topological relations between objects.” (Coventry & Garrod, 2004, p. 8)

Projective prepositions describe spatial relationships among objects, such as *above*, *below* or *to the left of*, including a direction in which one object (the *located object* or *trajector*) is located with respect to another object (the *reference object* or *landmark*). This work focuses on projective prepositions. From now on, I will use the terms *landmark* and *trajector*.

Projective prepositions need a reference frame in which they are interpreted. Consider the utterance “The ball is *to the right of* the chair”. This sentence can be understood in several ways depending on the used reference frame, see Figure 2.2. The use of different reference frames was the source of the confusion described in the introduction. In (Coventry & Garrod, 2004, p. 8, p. 92) three reference frames are listed: an *intrinsic reference frame*, a *relative reference frame* and an *absolute reference frame*.

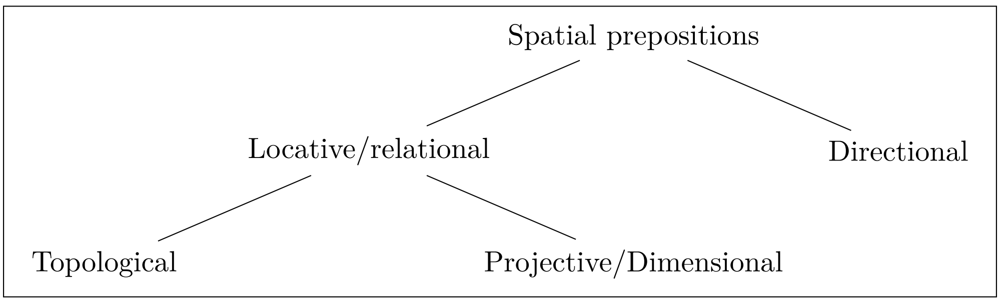


Figure 2.1: Classification of spatial prepositions. Adapted from (Coventry & Garrod, 2004, p. 7)

(a) Intrinsic reference frame centered in the landmark

(b) Relative reference frame centered in the observer

Figure 2.2: “The ball is *to the right of* the chair”, in bird’s-eye view with (a) an intrinsic reference frame and (b) a relative reference frame. The chair is schematized as an arrow.

The intrinsic reference frame is centered in the landmark. Considering the above mentioned utterance the trajector (the ball) would be located with respect to the landmark (the chair) as shown in Figure 2.2a.

The relative reference frame is centered in the viewpoint of an observer describing the scene. Based on this reference frame, the utterance would be apprehended the way it is depicted in Figure 2.2b.

The absolute reference frame is centered with respect to arbitrary, but fixed directions like cardinal directions (east/west) or gravity.

More on reference frames can be found in (Coventry & Garrod, 2004, p. 92-100).

The computational model of (Logan & Sadler, 1996, cited in Carlson and Logan (2005)) formalizes the apprehension of spatial terms by a process comprising the following steps: *spatial indexing*, *assigning directions in space* and *computing goodness of fit*.

Spatial indexing refers to the process of linking linguistic entities of an utterance (e.g., the chair in “The ball is to the right of the chair”) to perceived objects.

In the step assigning directions in space, a reference frame is fixed to the landmark and so-called *spatial templates* with the landmark as origin are computed. The spatial template of a spatial relation defines regions of acceptability around the landmark. Figure 2.3 shows a spatial template of *above* with brighter color meaning higher acceptability.

In the last step (computing goodness of fit) the spatial template which fits best to the location of the trajector is chosen as the adequate spatial relation between landmark and trajector.

In Carlson and Logan (2005) empirical evidence is reviewed showing that visual spatial attention is involved in spatial indexing and assigning directions to space. Especially, the construction of a spatial template is formalized in the *attentional vector sum* model (henceforth AVS) in which attention plays a central role (Regier & Carlson, 2001). Additional information on attention can be found in Chapter 3, the AVS model is explained in detail in Section 4.1.

Figure 2.3: The spatial template of *above* with an example landmark (toothbrush). Brighter color means higher acceptability of *above*.

2.1 Functional Aspects

Humans take functional aspects of objects into account when they have to decide whether a spatial preposition is appropriate to describe the spatial relationship between these objects or not. For instance, in (Carlson-Radvansky, Covey, & Lattanzi, 1999, experiment 1) a study is reported, in which participants had to place pictures of a toothpaste tube *above* a (picture of a) toothbrush. The toothpaste tube was not placed above the center of mass of the toothbrush, but toward the toothbrush bristles. The same, though less pronounced effect could be observed when the toothpaste tube was replaced with a tube of oil paint.

Several other studies (e.g., Carlson-Radvansky and Radvansky (1996); Coventry, Prat-Sala, and Richards (2001); Hörberg (2008), see Coventry and Garrod (2004) for more) give further evidence that both, geometric and functional aspects, are important for the production and comprehension of spatial terms.

Coventry and colleagues propose the *functional geometric framework* (Coventry et al. (2005); Coventry and Garrod (2004); Coventry et al. (2010)) as an explanation for the mechanisms underlying spatial term use. The functional geometric framework assumes two components which affect spatial language: *geometric routines* and *extra-geometric information*. Geometric routines only consider the geometric relations between objects. Extra-geometric information accounts for functional effects in spatial language.

This extra-geometric information may originate from two sources: *dynamic-kinematic routines* and *object knowledge*. Dynamic-kinematic routines compute the way two "objects (...) may interact with each other" (Coventry & Garrod, 2004, p. 55). An example of a dynamic-kinematic routine is what Coventry and colleagues call location control: If an object A is *in* another object B, then moving B will control the location of A.

Object knowledge is prior knowledge about the considered objects leading to an influence in the acceptance of spatial prepositions. For instance, (Coventry, Carmichael, & Garrod, 1994, cited in Coventry and Garrod (2004)) found a difference in preferred prepositions (*in* and *on*) just by using different labels for the landmark (*dish* or *plate*).

Object knowledge also refers to the situation in which the object is shown. If the spatial relation between a toothbrush and a toothpaste tube needs to be specified, the bristles of the toothbrush are the important functional part. However, if the toothbrush should be grabbed, the handle of the toothbrush becomes the important part.

The functional geometric framework aims to be a framework for all spatial prepositions. The AVS model only considers projective prepositions. Coventry and Garrod (2004) suggest that AVS might be the geometric routine for projective prepositions, since the AVS model performs well on geometric shapes (shown in Regier and Carlson (2001)).

In Carlson et al. (2006) the AVS model was extended to account for functional effects. This extension (fAVS) is presented in Section 4.1.2. However, (Coventry et al., 2010, p. 211) claim that "modifications of the AVS model proposed to integrate information regarding object function (...) are unable to account for the rating data, or the eye tracking data" they obtained. Thus, the relationship of fAVS and the functional geometric framework still needs to be clarified.

In the next chapter, I will provide an overview of attention, since attention is an integral component of the AVS model.

3 Attention

In this chapter, I will first introduce some basic aspects of attention and briefly introduce some influential theories of attention. Then, I will define visual attention more closely.

3.1 Attention in General

More than 100 years ago William James stated:

> Every one knows what attention is. It is the taking possession by the mind, in clear and vivid form, of one out of what seem several simultaneously possible objects or trains of thought. Focalization, concentration, of consciousness are of its essence. It implies withdrawal from some things in order to deal effectively with others, and is a condition which has a real opposite in the confused, dazed, scatterbrained state which in French is called *distraction*, and *Zerstreutheit* in German. (James, 1890, p. 403-404)

Nevertheless, research about the role of attention in perception and cognitive processes is still ongoing and even a growing research area.

Attention is often referred to as a mechanism that selects specific (task-relevant) input stimuli and ignores other (task-irrelevant) input, since cognitive resources for information processing are limited. Thus, information processing is enhanced at the attended stimulus. In other words, attention enhances information processing.

Consider the so-called "Cocktail-Party"-phenomenon: You are at a party, with, say, 50 guests. Surely, it is impossible to be in conversation with all of them. However, you are in a conversation with 4 or 5 guests in your vicinity. Even if it is quite loud because of other conversations around you and background music, you are able to follow your conversation.

This observation leads to the definition of attention as a filter, that excludes unimportant stimuli, so you can concentrate on important ones. (Broadbent, 1958, cited in Hagendorf, Krummenacher, Müller, and Schubert (2008)) developed a model formalizing this view. In this model, attention acts as a *filter* or a *bottleneck* in information processing. It is referred to as Broadbent's filter model of attention. All unimportant stimuli are ignored and not processed at all. This is a so-called *early selection* model, since attention selects certain stimuli before processing their meaning.

Again, consider being at the party. Suddenly, your name is called in a conversation you are not following. Even if you have no clue of what they are talking (because you filtered this conversation), you react to your name-calling. This would not be possible, if attention would select stimuli before further processing them, since you have to analyze the syllables of the called word to recognize them as your name.

Therefore, (Deutsch & Deutsch, 1963, cited in Hagendorf et al. (2008)) came up with a so-called *late selection* model. In this model, attention selects the important stimuli after an initial processing of all stimuli.

Another distinction of attention is also explicable with the cocktail-party phenomenon: *top-down* versus *bottom-up* attention. This distinction is related to, but different from early- versus late-selection.

The attention employed to the conversation in your vicinity is controlled by your goal to follow the conversation and thus also referred to as *top-down*, *voluntary*, *active*, *task-driven*, or *endogenous* attention.

The reaction on your name-calling is an example of attention that is also called *bottom-up*, *reflexive*, *involuntary*, *passive*, *stimulus-driven*, or *exogenous* attention: The stimulus grabs your attention because of its physical properties. For instance, *bottom-up* attention also happens when you drive by car and suddenly someone runs on the street.

For more detailed information regarding these theories of attention and their influence on modern theories, see for instance Hagendorf et al. (2008). An excellent overview combining psychological and philosophical considerations on attention is given by Mole (2013).

3.2 Visual Attention

In the previous section, attention was considered as a general mechanism. In fact, attentional mechanisms are widespread in perceptual and cognitive operations, see Chun, Golomb, and Turk-Browne (2011) for a recent taxonomy of attention. However, in this work visual attention is of particular importance. Thus, I will subsequently focus on visual attention.

Visual attention can be either *covert* or *overt*. During covert attention the eyes are not moved to the attended location while overt attention is visible through eye movements. These two types of visual attention are related:

> The consensus is that covert attention precedes eye movements, and that although the effects of covert and overt attention on perception are often similar, this is not always the case. (Carrasco, 2011, p. 1487)

Since computer vision systems face similar problems like the human vision system (particularly, selecting regions of interest among the vast amount of visual data), the computer vision community modeled visual attention – mainly focusing on overt visual attention. For recent overviews of the state of the art in computational models of visual attention see (Borji & Itti, 2013; Frintrop, Rome, & Christensen, 2010). These models are applied in computer vision systems, e.g., to reduce computational complexity. An overview of applications is given in Frintrop et al. (2010).

In the following, I will give a summary about psychological findings and theories of visual attention.

According to the recent review of visual attention by Carrasco (2011), three types of visual attention can be distinguished: space-based (spatial) attention, object-based attention, and feature-based attention.

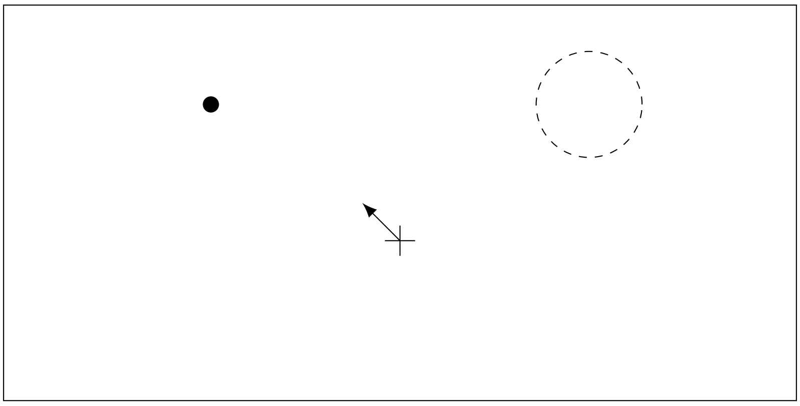


Figure 3.1: Posner's spatial cueing paradigm with a valid central cue (arrow), an invalid peripheral cue (dashed circle) and the target (filled circle). Observers must fixate the cross. Only one cue appears before the target shows up.

The canonical experimental paradigm to investigate spatial attention by directing it to a location and probe for the attentional distribution is Posner's spatial cueing paradigm (Posner, 1980, cited in Carrasco (2011)), visualized in Figure 3.1. In this paradigm, subjects first have to fixate the central region. Then, a central or peripheral cue is displayed for 40 ms (peripheral cue) or 150 ms (central cue) (Carrasco, 2011, p. 1490). A central cue appears in the central fixation area pointing to the cued area (e.g., an arrow) and a peripheral cue appears directly at the location where a target might show up.

Following this cue (and a so-called interstimulus intervall of 60 ms to 150 ms), a target appears either in the cued region or in the uncued region. In the former case, the cue was *valid*; in the latter case, the cue was *invalid*. Participants have to respond to this target as quickly as possible. If the target appears in the cued region, responses will be faster than if it appears in an uncued region. The explanation for this effect is the enhanced processing at the location of the cue.

According to these findings, spatial attention is described as a spotlight in Posner (1980), a zoom lens in Eriksen and Yeh (1985), or as a Gaussian gradient in Downing and Pinker (1985) (all cited in Carrasco (2011)).

Following these widely-used metaphors, spatial attention consists of a point of highest attention (*attentional focus*) and a circumscribed region around this focus, where attention gradually drops off. The size of the attended region is adjustable voluntarily, depending on task-demands. In the attended region, processing of visual stimuli is enhanced; outside this region, information is inhibited. The recent review Carrasco (2011) covers spatial attention.

Object-based attention refers to attention deployed on objects rather than space or features. A seminal paradigm to investigate object-based attention was created by (Egly, Driver, & Rafal, 1994, cited in Chen (2012)). The stimulus Egly et al. (1994) used is depicted in Figure 3.2. A cue is placed at the end of one of the rectangles. The target

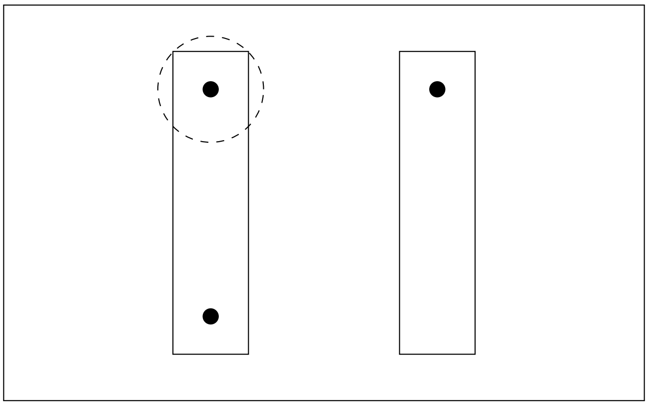


Figure 3.2: Sample stimulus adapted from Egly et al. (1994) to probe for object-based attention. Cued location is on the top left (dashed circle). Three target locations are shown (filled circles); two of them are invalid, but with the same distance from the cue.

following the cue can appear at three possible locations: at the cued location, at the other end of the rectangle or at the other rectangle.

Main results were that responses to a target showing up in the same object at an invalid location were faster than responses to a target at an invalid location in the other rectangle. Because the distances between the cue and the two invalid target areas are the same, this can not be explained by pure spatial attention. Note, however, that what is perceived as an object is still an open question, see e.g. Chen (2012). Recent reviews of object-based attention are provided by Chen (2012); Scholl (2001).

Feature-based attention is attention focusing on features in dimensions other than space such as color, orientation, or motion. Feature-based attention acts on the whole visual field and is not constrained by the spatially attended region. For example, attention to the color red enhances reactions to red colored objects, even if they do not appear within the "spotlight" of spatial attention. Again, more details on feature-based attention can be found in Carrasco (2011).

3.2.1 Interaction between different types of visual attention

There is evidence that the three attentional mechanisms described above guide our visual attention. However, in normal life we perceive the visual world as one coherent representation and are not aware of different attentional mechanisms. In Vecera and Behrmann (2001), the *biased competition* account developed in (Desimone & Duncan, 1995, cited in Vecera and Behrmann (2001)) is adapted to combine spatial attention and object-based attention.

The biased competition model was developed to explain visual search tasks, where a subject needs to find a target (e.g., a red rectangle) in a set of distractors (e.g., black rectangles). If the target is unique in most of its features (red vs. black), it will *pop out* and humans will be able to quickly react to the visual search task by using a so-called *feature-search*. If, however, the target shares features with the distractors (e.g., finding a vertical red rectangle in a set of vertical black and horizontal red rectangles), the reaction time to find the target (with a so-called *conjunction search*) will increase with the number of distractors.

The biased competition model proposes that different visual stimuli in the visual field compete for attention and that the attentional selection is biased both by *bottom-up* mechanisms (e.g., color) and *top-down* mechanism (e.g., the need to find a specific target).

In Vecera and Behrmann (2001), it is used as a framework to combine spatial, feature-based, and object-based attention. Recent studies investigated the interactions between the three attentional systems (Kravitz & Behrmann, 2011; Soto & Blanco, 2004).

3.2.2 Divided Visual Spatial Attention

The spotlight and zoom lens metaphors mentioned above assume that spatial attention has a single indivisible focus point. However, it is debated controversially, if spatial attention might be divided, resulting in a multimodal attentional distribution. There are some studies showing that humans are able to split their attention, however, as Jans, Peters, and De Weerd (2010) claim, they all have methodological pitfalls. As a consequence, they propose four methodological criterias to avoid these problems. Nevertheless, Cave, Bush, and Taylor (2010) argue that these criterias are too strict. Newest research is still dealing with this question, e.g., Feng and Spence (2013); Jefferies, Enns, and Di Lollo (2013).

This work also aims at giving evidence either to a unifocal or a multifocal attentional distribution. This evidence is gained by applying computer simulations of cognitive models assuming specific attentional distributions to empirical data.

3.2.3 Research Questions

Now, as the background of the present work is formulated, it is time to more precisely define my research questions:

The AVS model, mentioned in Chapter 2 and presented in more detail in Chapter 4, tries to explain the mechanisms behind the acceptance of spatial prepositions by using visual spatial attention. Since spatial language is affected by functional aspects, the authors of Carlson et al. (2006) proposed an functional extension to the AVS model (fAVS) that takes the function of objects into account.

As shown in Section 4.1.2, the fAVS model makes the assumption of divided visual spatial attention. Since this assumption is controversially debated (see Section 3.2.2), alternative extensions without this assumption, presented in Section 4.2, were developed. Especially, the answer to the following research question is of interest:

Are the alternative functional extensions to the AVS-model able to gain more or less support from empirical data in contrast to the original extension? Or, to put it differently: Is divided attention a necessary assumption?

4 Models

In this chapter, I will introduce the before mentioned AVS model and its functional extension fAVS in detail (Section 4.1). I will discuss the attentional distribution fAVS is assuming in Section 4.1.2 and develop alternative extensions in Section 4.2.

4.1 AVS

The *attentional vector sum* model (AVS) was proposed by Regier and Carlson (2001) as a cognitive model of the acceptance of spatial prepositions. In its first version, the AVS-model does not consider functional parts of objects. In Regier and Carlson (2001) it was tested on geometric shapes like rectangles and triangles. In the following section, I will give an introduction on how the model works. In Section 4.1.2, I will present fAVS, an extension to this model which takes functional parts of landmarks into account.

4.1.1 Original AVS

The input of the AVS model is a landmark-object, a trajector position and a spatial preposition (e.g., *above*). The output consists of an acceptance rating on a given scale. If AVS returns a low rating, the spatial term *above* is not considered to adequately describe the spatial relation between landmark and trajector. On the other hand, if the rating is high, the AVS model predicts that most observers accept *above* as an appropriate description of the scene.

By that, AVS computes the spatial template of a spatial relation, i.e. the regions of acceptability of a spatial relation around a landmark (see Figure 2.3 on page 9 for an example template).

To obtain the rating AVS computes two components: A height component and an angular component.

The height component is a value between 0 and 1. It will take the value "1 if the trajector is well above all landmark top points and 0 if it is well below all landmark top points" (Regier & Carlson, 2001, p. 274). The top of the landmark is hereby defined as all points of the landmark, where one cannot find a point with the same x-value, but a higher y-value. The computation of the height component in detail:

$$\text{height}(y) = \frac{\text{sig}(y - \mathit{hightop}, \mathit{highgain}) + \text{sig}(y - \mathit{lowtop}, 1)}{2} \tag{4.1}$$

The sig-function is defined as:

$$\text{sig}(x, \mathit{gain}) = \frac{1}{1 + \exp(-x \cdot \mathit{gain})} \tag{4.2}$$

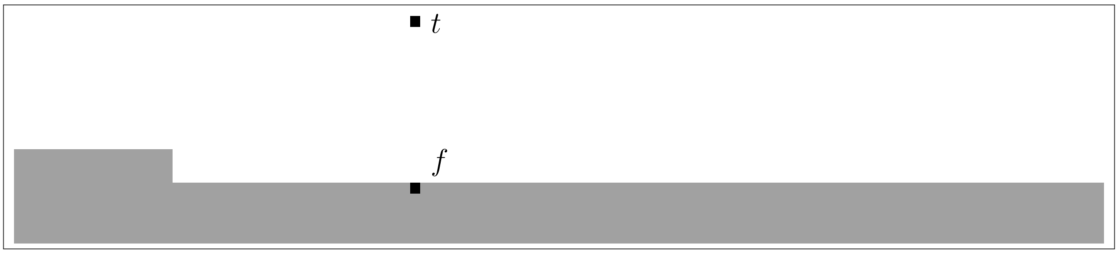


Figure 4.1: An example landmark (toothbrush) with trajector t and its resulting focus f which the AVS model assumes.

hightop is the y-value of the highest point on top of the landmark, *lowtop* is the y-value of the lowest point on top, and *highgain* is a free parameter.

In a first step to obtain the angular component, an attentional focus f with respect to the location of the trajector is defined. Its center is the vertically aligned point on top of the landmark. In case the trajector is not vertically aligned with the landmark, the attentional focus lies at the point that is closest being so, i.e., the closest edge of the landmark. Figure 4.1 shows an example trajector t and the attentional focus f which the model assumes.

Every point i of the landmark receives a certain amount of visual spatial attention. The attention is highest in the attentional focus f and decreases exponentially following formula 4.3 in which d_i denotes the euclidean distance between landmark point i and the focus f, σ denotes the euclidean distance between focus f and trajector t, and λ is a free parameter:

$$a_i = \exp\left(\frac{-d_i}{\lambda \cdot \sigma}\right) \tag{4.3}$$

This exponential decay function is inspired by empirical studies of visual spatial attention by (Downing & Pinker, 1985, cited in Regier and Carlson (2001)).

Next, a vector $\vec{v_i}$ is rooted at every landmark point i, pointing to trajector t and weighted by the amount of attention a_i at landmark point i. Figure 4.2 visualizes this process using the same example trajector t as in Figure 4.1. All these vectors are summed up to give one vector:

$$\overrightarrow{direction} = \sum_{i \in LM} a_i \cdot \vec{v_i} \tag{4.4}$$

The use of a vector sum is inspired by empirical findings "that in several neural subsystems, overall direction is represented as the vector sum of a set of constituent directions" (Regier & Carlson, 2001, p. 277). Empirical support is coming from studies that investigated rhesus monkey motor cortex Georgopoulos, Schwartz, and Kettner (1986), saccadic eye movements Lee, Rohrer, and Sparks (1988), and motion perception Wilson and Kim (1994) (all cited in Regier and Carlson (2001)).

Moreover, Lipinski, Schneegans, Sandamirskaya, Spencer, and Schöner (2012) present a neural dynamic model that is consistent with empirical data and "highly compatible

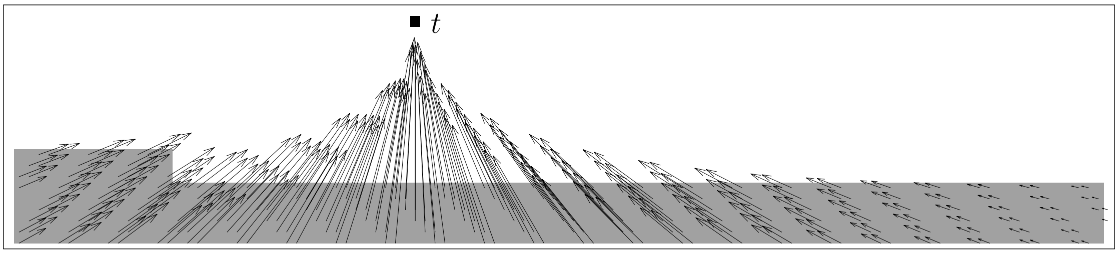


Figure 4.2: An example trajector t and resulting vectors $\vec{v_i}$ weighted by attention a_i.

with the AVS model, showing how the neural population coding of location central to AVS can support behavioral flexibility when extended to the level of neural dynamic processes" (Lipinski et al., 2012, p. 1508).

To obtain a rating from the vector sum, a linear function $\mathrm{g}(a)$ is used. $\mathrm{g}(a)$ maps the angular deviation $angle = \measuredangle(up, \overrightarrow{direction})$ from upright vertical (in case of *above*) to the rating scale, with more deviation resulting in lower ratings:

$$\mathrm{g}(angle) = slope \cdot angle + intercept \tag{4.5}$$

Here, *slope* and *intercept* are free parameters.

The height component and the angular component are multiplied to get a final rating. Thus, the complete AVS-model can be written as:

$$above(t) = \mathrm{g}\left(\measuredangle\left(up, \sum_{i \in LM} a_i \cdot \vec{v_i}\right)\right) \cdot \mathrm{height}(y_t) \tag{4.6}$$

4.1.2 fAVS

The original AVS model performs well on geometric shapes as reported in Regier and Carlson (2001). But, as already discussed in Section 2.1, humans also consider functional aspects of objects. Therefore, Carlson et al. (2006) proposed fAVS, a functional extension to the AVS-model. The idea behind this extension is that a functional part attracts more attention such that the amount of attention at every point i lying in the functional part is increased:

$$A_i = \begin{cases} a_i \cdot (1 + \varphi) & \text{if } i \text{ lies in functional part} \\ a_i & \text{else} \end{cases} \tag{4.7}$$

Here, a_i denotes the amount of attention at point i, as defined in equation (4.3) and φ is a free parameter. In the fAVS model, A_i is used instead of a_i to weight each vector v_i.

In Carlson et al. (2006), three values were suggested for φ:

$\varphi = 2$	strong functional interaction between landmark and trajector
$\varphi = 1$	weak functional interaction between landmark and trajector
$\varphi = 0$	no functional interaction (i.e. the original model)

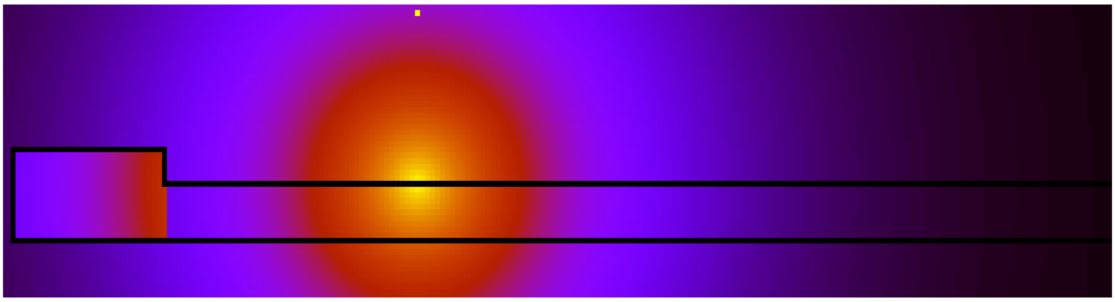


Figure 4.3: Bimodal attentional distribution when using fAVS. Toothbrush bristles are defined as functional part. Brighter color means higher amount of attention. Trajector is positioned at the yellow dot. Borders of landmark are colored black.

Again, empirical findings support the assumption of higher amount of attention on functional parts. (Lin & Murphy, 1997, cited in Carlson et al. (2006) and Regier, Carlson, and Corrigan (2005)) demonstrated that people are quicker to detect a missing part of an object if this part was functional than if the missing part was non-functional.

To further support this, Carlson et al. (2006) conducted an experiment showing that functional parts of objects indeed act like spatial cues in Posner's spatial cueing paradigm (see Section 3.2) – i.e. a functional part attracts spatial attention and this is measurable by faster response times due to facilitated processing at the functional part.

Attentional Distribution

Figure 4.3 shows an exemplary attentional distribution fAVS assumes. It can be seen that fAVS leads to a bimodal attentional distribution[1], since the functional part receives more attention than its surroundings.

By this, fAVS assumes that humans are able to divide their visual spatial attention. As already discussed in Section 3.2.2, it is controversially debated whether this is possible or not. Therefore, I subsequently will present three different functional extensions of the AVS model without the implicit assumption of a multimodal attentional distribution.

4.2 Alternative functional extensions for AVS

In this section, I present three alternative functional extensions to the AVS-model. In the first two extensions (Section 4.2.1 and 4.2.2), the location of the attentional focus is varied; in the last extension (Section 4.2.3), an attentional switch is proposed. Note that for all extensions the key idea – functional parts attract attention – stays the same, whereas there is no implicit assumption of a multimodal attentional distribution.

[1]except in those cases, where the attentional focus f lies in the functional part

Figure 4.4: An example trajector t and its resulting focus mf using the *move focus* extension. The functional part is colored in dark gray.

4.2.1 Move Focus

In this extension, the location of the attentional focus f, i.e. the point with the greatest amount of attention, is changed. In the original model, the attentional focus f is the “point on the landmark top that is vertically aligned with the trajector or closest to being so aligned” (Regier & Carlson, 2001, p. 277). This extension also starts with this focus point, but another step is added . The focus is moved into the direction of the functional part. The farther away the focus is from the functional part, the more it moves toward the functional part.

To compute the new focus the leftmost and rightmost point on the functional top are used, where the functional top is defined like the top of the landmark in the original AVS model (see explanation to equation (4.1)).

More precisely, the new focus $mf = (mf_x, mf_y)$ is derived from the original focus $f = (f_x, f_y)$, the leftmost point on top of the function $ltf = (ltf_x, ltf_y)$ or the rightmost point on top of the function $rtf = (rtf_x, rtf_y)$ as follows:

$$mf = \begin{cases} \left(\frac{f_x + ltf_x}{2}, \frac{f_y + ltf_y}{2}\right) & \text{if } f \text{ is to the left of the functional part} \\ \left(\frac{f_x + rtf_x}{2}, \frac{f_y + rtf_y}{2}\right) & \text{if } f \text{ is to the right of the functional part} \\ (f_x, f_y) & \text{if } f \text{ lies in the functional part} \end{cases} \tag{4.8}$$

Figure 4.4 shows the focus point mf resulting from the same example trajector t used to illustrate the original model. Note that with this extension the location of the focus point may not lie inside the landmark, as can be seen in Figure 4.4. Henceforth, this extension will be referred to as the *move focus* extension.

4.2.2 Focus only at Functional Part

This extension again changes the location of the focus point. This time the focus always lies on the functional part. The way the focus point is chosen is almost the same as in the original model. The only change is that the top of the functional part is used instead of the whole top of the landmark. Figure 4.5 shows an exemplary resulting focus point. From now on, I will refer to this extension as the *focus only at function* extension.

Figure 4.5: An example trajector t and its resulting focus f using the *focus only at function* extension. The functional part is colored in dark gray.

4.2.3 Attentional Switch

In this last alternative extension, an attentional switch is assumed. First, the attentional focus lies on the functional part of the landmark and then the landmark is attended as if it had no functional part. In greater detail, the extension consists of the following steps:

1. One vector sum is computed as in the AVS model, but with the focus f_1 chosen like in the *focus only at function* extension (i.e., the focus lies on the functional part).
2. A second vector sum is computed, but this time the attentional focus f_2 is the one from the AVS model, that is, the landmark is handled as if it was not containing any functional part.
3. The deviation from upright vertical (in the case of *above*) is measured for both vectors.
4. Both deviations are averaged to get a final value for the angular component.

Figure 4.6 shows the two vectors as defined in step 1 (left vector) and step 2 (right vector) in light gray. The vector drawn in solid black is the vector with the average deviation from upright vertical, as stated in step 4. However, note that, to avoid visual clutter, the starting points and lengths of the vectors have been modified. In the remainder, this extension will be referred to as the *attentional switch* extension.

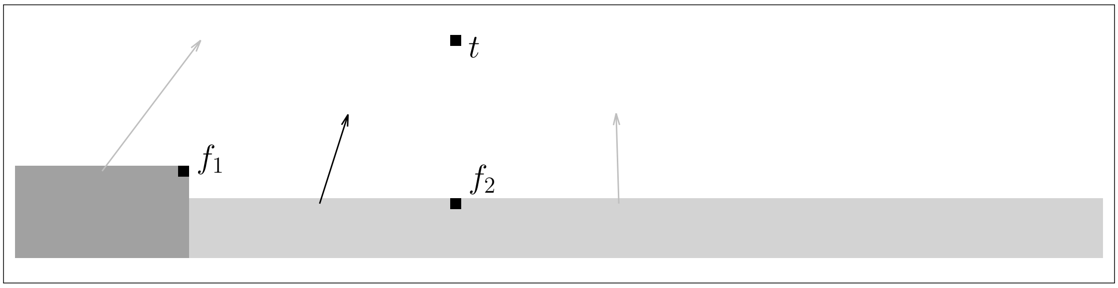


Figure 4.6: An example trajector t, two foci f_1 and f_2 and the resulting vectors according to the *attentional switch* extension. The functional part is colored in dark gray.

5 Method and Data

In the previous chapter, I introduced five slightly different models that calculate acceptability ratings of spatial prepositions. In the following, I will compare these models with respect to two main questions:

1. Are the alternative functional extensions to the AVS model able to gain more or less support from empirical data in contrast to fAVS? Or, to put it differently: Is divided spatial attention a necessary assumption?

2. Is any of the functional extensions able to better account for functional effects in empirical data than the AVS model? In other words, do the functional extensions appropriately capture functional aspects of spatial language?

In this chapter, I will first describe the method used to evaluate the models. Then, I will present the data used in the model simulations. In the next chapter, I will present the results of the model comparison and discuss them.

5.1 Method

The AVS model and its variations were implemented in `C++` (for more details on the implementation see Appendix A). The program is able to compute a Goodness-of-Fit value (GOF) – a measure indicating how close a model can fit to data.

To compute the GOF-value all model parameters were estimated by minimizing the Root Mean Square Error (RMSE) to get the tightest fit. The RMSE is defined as follows, with n being the number of data points:

$$RMSE = \sqrt{\frac{1}{n}\sum_{i}^{n}(data_i - modelOut_i)^2} \tag{5.1}$$

The GOF gives a first approximation on how well a model can simulate empirical data. If none of the compared models is able to fit to the data, there would be no need to further consider this model.

To estimate model parameters the *simulated annealing* method is used. In comparison to simple gradient descent methods, simulated annealing has the advantage to not get stuck in local minima by temporarily accepting worse values. Simulated annealing is a special case of the *metropolis algorithm*, named after Metropolis, Rosenbluth, Rosenbluth, Teller, and Teller (1953). The metropolis algorithm is a Markov Chain Monte Carlo approach (MCMC). More on the theoretical background can be found in Madras (2002).

The procedure of *simulated annealing* is given in Algorithm 1. Starting values of the AVS parameters were set to parameters in (Regier & Carlson, 2001, Table 1, Logan&Sadler fit).

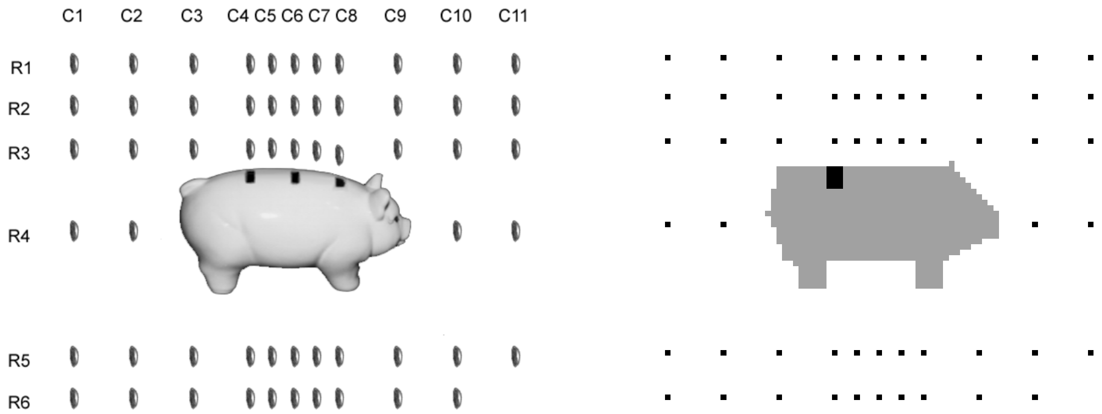
Note, however, that some of the model parameters were constrained as follows:

$$highgain \in [0, 10]; \lambda \in [0, 5]; \varphi \in [0, 2]$$

This was done to (i) exclude cognitive implausible parameter values and (ii) stabilize the parameter estimation algorithm. Parameters of the algorithm were initialized as follows: $temperature = 0.25$, $iterations = 500$, $coolingPeriod = 300$.

Algorithm 1: Simulated annealing algorithm in pseudo-code

```
for each parameter p do
|   p = starting value;
end
oldRMSE = computeRMSE(data, parameters);
bestRMSE = oldRMSE;
for i = 1 to iterations do
|   for j = 1 to coolingPeriod do
|   |   for each parameter p do
|   |   |   newP = sampleNormalDistribution(μ = p, σ = temperature);
|   |   end
|   |   newRMSE = computeRMSE(data, newParameters);
|   |   if newRMSE < oldRMSE then
|   |   |   for each parameter p do
|   |   |   |   p = newP;
|   |   |   end
|   |   |   if newRMSE < bestRMSE then
|   |   |   |   bestRMSE = newRMSE;
|   |   |   end
|   |   else
|   |   |   // worse result but still accept with probability accept
|   |   |   accept = sampleUniformDistribution(0, 1);
|   |   |   if accept ≤ exp(−(newRMSE−oldRMSE)/temperature) then
|   |   |   |   for each parameter p do
|   |   |   |   |   p = newP;
|   |   |   |   end
|   |   |   end
|   |   end
|   |   oldRMSE = newRMSE;
|   end
|   temperature = 0.99 · temperature;
end
```



(a) Landmark, trajector positions, and functional parts used in Carlson-Radvansky et al. (1999). In the experiment, only one coin and one slot were shown at the same time. (Image source: (Carlson-Radvansky et al., 1999, p. 519))

(b) Schematized piggy bank with slot in the back used by the `C++`-implementation to obtain model evaluation results.

Figure 5.1: Landmark and trajector positions for Carlson-Radvansky et al. (1999) data

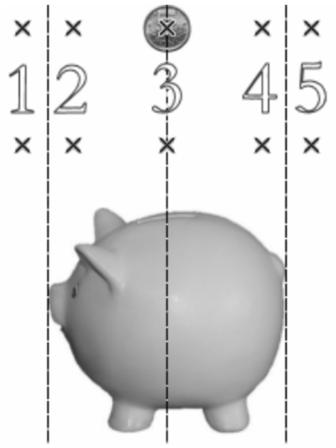
5.2 Data

To compare the AVS model with the presented variations, I used data from (Carlson-Radvansky et al., 1999, experiment 2) and Hörberg (2008)[1]. For both data sets the experimental setup was the following: Participants were shown a landmark and a trajector at different positions around the landmark. For each trajector (shown at a specific position), participants had to rate the appropriateness of the sentence "The [trajector] is [spatial-preposition] the [landmark]" on a scale from 1 to 7, with 1 being lowest acceptance and 7 highest acceptance.

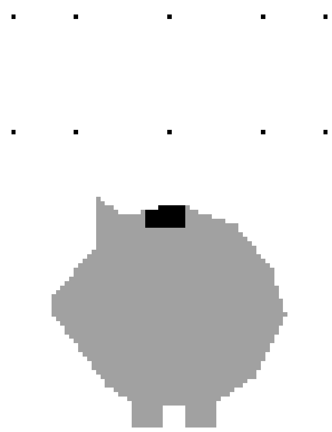
5.2.1 Carlson-Radvansky et al.

Figure 5.1a shows the landmark and trajector positions used in (Carlson-Radvansky et al., 1999, experiment 2), Figure 5.1b shows the schematized landmark used in the simulations. As can be seen in Figure 5.1a the functional part (i.e. the coin slot) was altered as an experimental condition to compare the acceptability of spatial prepositions in relation to the location of the functional part. For each of the three slot positions ratings were obtained for all 58 trajector positions from each participant. The considered spatial term was *above*.

[1] I thank Thomas Hörberg for sharing his data.



(a) Landmark, trajector positions, and functional part of the piggy bank in Hörberg (2008). (Image source: (Hörberg, 2006, p. 31))



(b) Schematized piggy bank and trajector positions used for simulation. Functional part is colored in black.

Figure 5.2: Piggy bank and trajector positions for Hörberg (2008) data

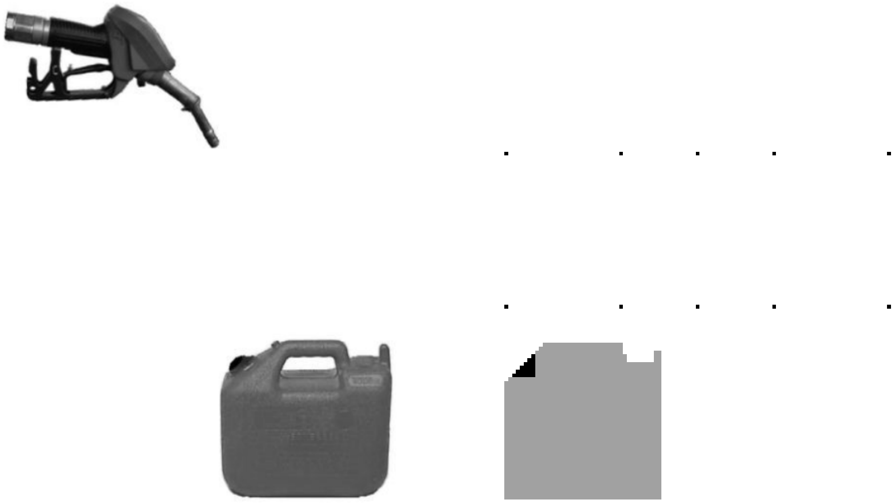
5.2.2 Hörberg

In Hörberg (2008), experiments are presented that investigated the Swedish spatial prepositions *ovanför, över, nevanför* and *under* (corresponding to the English prepositions *above, over, below* and *under*) with respect to their acceptability when influenced by a functional relationship between landmark and trajector.

The main purpose of the experiments in Hörberg (2008) was to find differences in the way different spatial prepositions are influenced by functional parts. It is shown in (Coventry et al., 2001, cited in Hörberg (2008)) that the English prepositions *over* is influenced more strongly by functional relationships than *above*. Corresponding results were found for *under* and *below*. This effect could also be observed for the Spanish prepositions *encima de* (stronger influenced) and *sobre* ((Coventry & Guijarro-Fuentes, 2004, cited in Hörberg (2008))).

As the original AVS model and the experiment 2 in Carlson-Radvansky et al. (1999) only considers *above*, I only used the corresponding Swedish preposition *ovanför*. However, it might be interesting to see how the AVS extensions can mirror the *över*-data, since in Hörberg (2008) this preposition is found to be more influenced by functional relationships.

The functional interaction between landmark and trajector in Hörberg (2008) is divided in two types: *center-of-mass aligned* and *center-of-mass deviant* interactions. Functional interactions of the first type are fulfilled when the center of mass of the trajector



(a) Landmark, trajector positions and functional part of the petrol canister in Hörberg (2008). (Image Source (Hörberg, 2006, p. 74))

(b) Schematized petrol canister and trajector positions used for simulation. Functional part is colored in black.

Figure 5.3: Petrol canister and trajector positions for Hörberg (2008) data

is above the landmark in a strict geometric way (e.g., coin over piggy bank, see Figure 5.2a). *Center-of-mass deviant* interactions are fulfilled when the center of mass of the trajector is to the left or to the right of the landmark (e.g., ketchup bottle over hot dog or Figure 5.3a).

To evaluate the AVS model and its extensions with both types of functional interaction, I chose one landmark-trajector pair for each type, shown together with their corresponding polygons used in the simulations in Figures 5.2 and 5.3. The piggy bank with its trajector (a coin) is an example of a *center-of-mass aligned* functional interaction, the petrol can with its trajector (a gas pump handle) is an example of a *center-of-mass deviant* interaction.

The functional part of most of the other landmarks used in Hörberg (2008) spanned over the whole top of the landmark – I did not use these landmarks, because all AVS extensions would have behaved the same for such landmark objects.

Since the AVS model represents trajectors only as a point, but the trajectors were quite big objects in the experiments, I had to decide, which trajector positions to use in the simulation. I used the center of mass of the coin trajectors for the data from both Hörberg (2008) and Carlson-Radvansky et al. (1999). This seems reasonable, since a coin is a small trajector and the functional interaction shown in Figure 5.2 is a *center-of-mass aligned* interaction.

Figure 5.3 shows an example of a *center-of-mass deviant* functional interaction. I decided to use the positions of the functional important parts of the trajector (i.e. the bottom right of the gas pump handle in Figure 5.3a) as trajector positions in the simulation and not the center of mass positions.

Every landmark in Hörberg (2008) was tested with a prototypical trajector (e.g., coffee mug and a sugar cube) and an aprototypical trajector (e.g., coffee mug and an ice cube). In this work only prototypical trajectors are used.

6 Results

The results presented in this chapter were obtained by applying the method described above to the data also presented in the previous chapter.

The description of the results follows the order of the data-sets described in Chapter 5: First, I will present and discuss the results for the Carlson-Radvansky et al. (1999) data, I will then go on to the results for the Hörberg (2008) data.

6.1 Goodness of Fit: Carlson-Radvansky et al.

As can be seen in Figure 6.1a, all models are able to tightly fit the data from Carlson-Radvansky et al. (1999). In comparison, the extensions *move focus* and *focus only at function* are the worst. The *attentional switch* extension results in neither the best nor the worst fit. The fAVS model fits the data best, but interestingly, the AVS model fits nearly equally well – without taking any functional parts of the landmark objects into account.

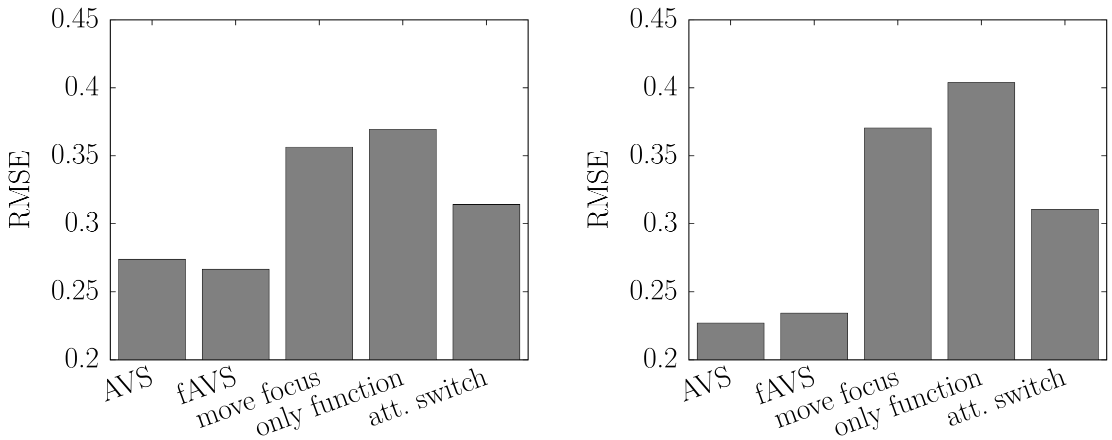
One reason for this might be that the data set contains a lot of trajector positions far away from the functional parts of the landmark (see Figure 5.1a). In addition, the functional effects in the data might be rather small. To increase the functional effect, I chose a *functional subset* of data points which only consists of the trajectors in columns C2-C9 and rows R1-R3 (see Figure 5.1a) assuming a stronger influence of the location of the functional part for this subset of data points.

The GOF results for the functional subset can be found in Figure 6.1b. As can be seen, the models almost show the same behavior when using the functional subset in comparison to the whole data set. Therefore, the possible difference in the functional effects in both data sets does not seem to be the main reason for the behavior of the models.

6.1.1 Parameter Values

Table 6.1 lists the parameter values leading to the best fit shown in Figure 6.1a. The best value for φ in fAVS is 0.6804. In terms of suggested φ values in Carlson et al. (2006) this indicates a weak functional interaction between landmark and trajector.

The *focus only at function* extension seems to need a high value for the attentional width parameter λ. To cover the whole landmark, the attention needs to be more widened than in all other extensions, since the attentional focus lies only at the functional part. Taking this together with the bad GOF value, the *focus only at function* extension seems to overestimate the functional part of the landmark.



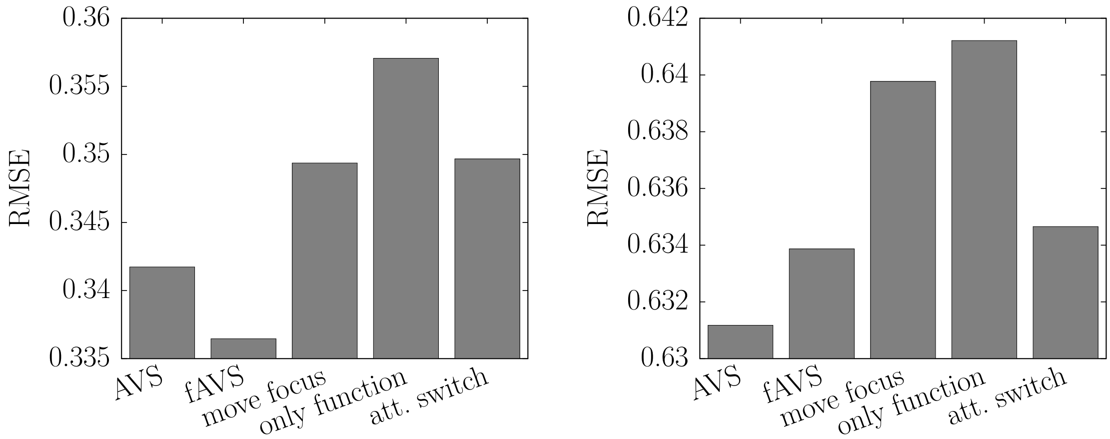
(a) GOF results for all positions

(b) GOF results for the functional subset

Figure 6.1: GOF results for Carlson-Radvansky et al. (1999)

Table 6.1: Best parameters to fit all positions corresponding to Figure 6.1a

Param.	AVS	fAVS	move focus	only function	att. switch
λ, att. width	0.3509	0.3182	0.7577	2.6720	0.8855
φ, func. strength	–	0.6804	–	–	–
slope	-0.0067	-0.0070	-0.0076	-0.0062	-0.0068
intercept	0.9698	0.9562	1.0592	1.0308	1.0315
highgain	7.2562	3.2024	7.7712	5.6501	8.6856



(a) GOF results for piggy bank

(b) GOF results for petrol canister

Figure 6.2: GOF results for Hörberg (2008)

6.2 Goodness of Fit: Hörberg

The GOF results for the data in Hörberg (2008) are depicted in Figure 6.2. The qualitative pattern is the same as for the data in Carlson-Radvansky et al. (1999): All in all, the AVS and fAVS models give the best fits, my extensions result in comparably poor GOF values with the *focus only at function* extension being the worst. Note, however, that the petrol canister data results in overall worse GOF values in comparison to the other data sets. One main reason for this could be the simplification of the trajector to a single point and the type of functional interaction (*center-of-mass deviant*).

The data sets from Hörberg (2008) were not split further into functional subsets, since they are considerably smaller than those from Carlson-Radvansky et al. (1999).

6.2.1 Parameter Values

The best parameter values resulting in the GOF values depicted in Figure 6.2a are listed in Table 6.2. Again, the parameter that reflects the functional strength in the fAVS model, $\varphi = 0.2805$, suggests a quite small functional interaction between landmark and trajector. Since my extensions are not able to account for different functional strengths, this might be a reason for their poor behavior.

Apart from that, the proposed extensions seem to need a greater attentional width λ than (f)AVS.

6.3 Simple hold-out

As outlined by Roberts and Pashler (2000) and Pitt and Myung (2002) a good fit is necessary, but not sufficient for a "good" model. A model might yield good fits just because of its complexity. Therefore, I also provide results obtained by the *simple hold-*

Table 6.2: Best parameters to fit the piggy bank corresponding to Figure 6.2a

Param.	AVS	fAVS	move focus	only function	att. switch
λ, att. width	1.8172	0.5420	4.4500	3.3878	4.0788
φ, func. strength	–	0.2805	–	–	–
slope	-0.0112	-0.0125	-0.0108	-0.0111	-0.0108
intercept	0.9350	0.9334	0.9368	0.9473	0.9368
highgain	6.5879	0.5880	0.6557	0.4893	5.0980

out (SHO) method. This method controls for the complexity of models by evaluating their ability to generalize to new data points and performs well compared to other model comparison methods (Schultheis, Singhaniya, & Chaplot, 2013).

The key idea of *simple hold-out* is to use only a part of the data to estimate parameters (or: train the model) and to predict the remaining data with these parameters (or: test the model). This is done several times using different partitions of the data. The RMSE of the prediction is saved for each iteration. In the end, the median of all prediction errors is used as an evaluation measure. The lower this median prediction error, the better the model is able to predict unseen data.

Algorithm 2 shows this procedure in pseudo-code. I used the following parameter values for the data in Carlson-Radvansky et al. (1999): $amountOfTrainingData = 70\%, iterations = 101$. I chose more iterations for the data sets from Hörberg (2008) ($iterations = 501$), since these are smaller.

Algorithm 2: *Simple hold-out* algorithm in pseudo-code

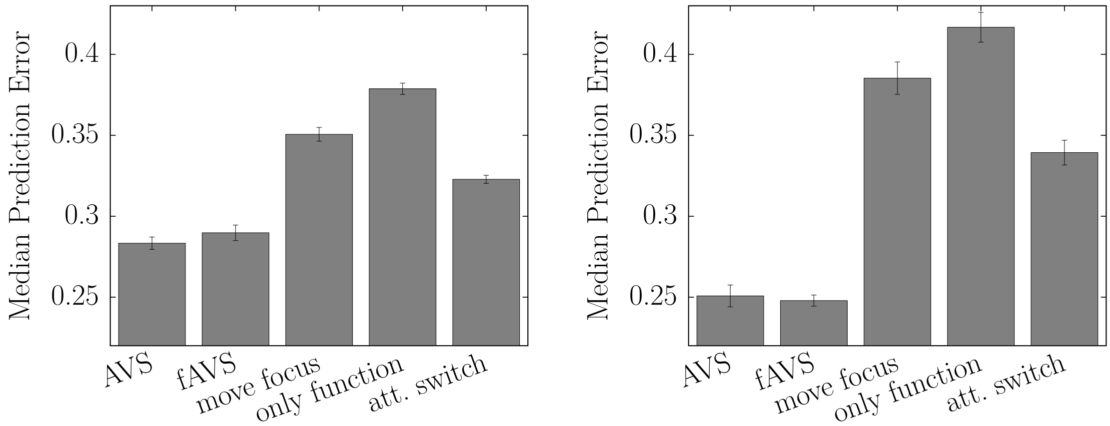
```
for i = 0 to iterations do
    trainingData = pickRandom(allData, amountOfTrainingData);
    fittedParameters = parameterEstimation(trainingData);
    testData = allData − trainingData;
    predictionErrors[i] = computeRMSE(testData, fittedParameters);
end
return median(predictionErrors)
```

6.3.1 Carlson-Radvansky et al.

The SHO results for the data from Carlson-Radvansky et al. (1999) are shown in Figure 6.3. The shown error bars are the bootstrap standard error estimates computed as stated in (Efron & Tibshirani, 1993, p. 47) using 100,000 bootstrap samples.

Looking closer to the behavior of the models by using the SHO method leads to similar results as already obtained through the GOF: My proposals behave worse than (f)AVS.

Still, AVS is able to mirror the data equally well as fAVS without taking functional



(a) SHO results for all positions

(b) SHO results for the functional subset

Figure 6.3: SHO results for Carlson-Radvansky et al. (1999)

parts into account. Maybe the functional aspects contained in the data are too small so that fAVS is acting like AVS by using small values for φ? It might also be, that fAVS does not capture the underlying cognitive mechanism appropriately.

6.3.2 Hörberg

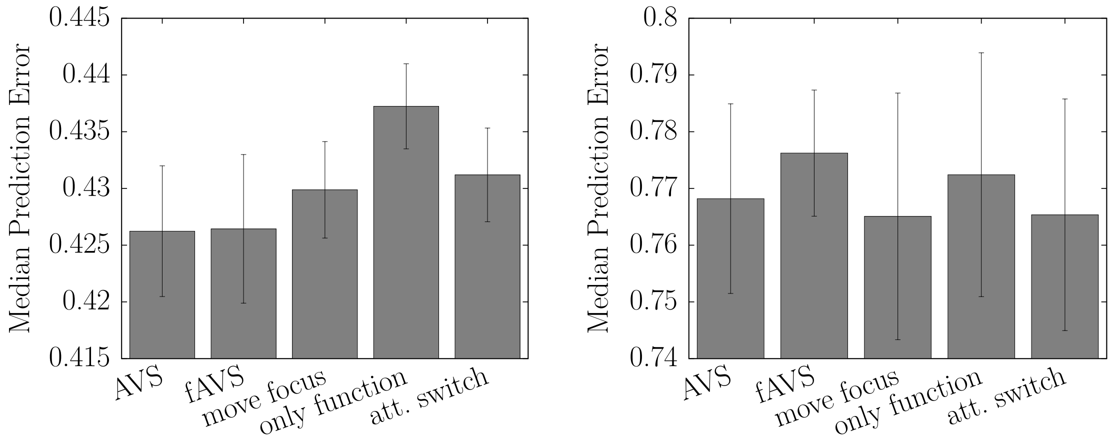
The SHO results for the data sets from Hörberg (2008) are depicted in Figure 6.4; again, error bars show the bootstrap standard error estimates.

These results are rather difficult to interpret, since the standard error is pretty big. Even using five times more iterations in the SHO method for the Hörberg (2008) data than the Carlson-Radvansky et al. (1999) data yields these big errors. However, ignoring the big error for now, the results for the piggy bank (Figure 6.4a) support the trend found so far.

The results for the petrol canister (Figure 6.4b) cannot favor any model. Indeed, they seem fairly chaotic. Again, this might be the case because all models simplify the trajector and seem not able to account for *center-of-mass deviant* interactions. This provides further evidence that the shape and the functional parts of the trajector are more important than currently assumed in the AVS model. An important step toward improving the AVS model should, therefore, consist of a better implementation of how shape and functional parts of the trajector are taken into account (see also discussion in Hörberg (2008)).

6.4 Modified Models

The results obtained so far give evidence that visual spatial attention can be split. However, AVS is able to account as well as fAVS for the empirical ratings. This might be because of rather weak functional effects in the used data.



(a) SHO results for piggy bank
(b) SHO results for petrol canister

Figure 6.4: SHO results for Hörberg (2008)

The fAVS model has a parameter that controls the functional strength. In fact, AVS is nested in fAVS – with $\varphi = 0$, fAVS becomes the same model as AVS. To put it differently, fAVS is able to gradually account for the functional strength. However, my proposed extensions can not weight the functional strength. Since the data seem to contain only small functional effects this could be a reason why the alternative extensions are not able to mirror the data as well as (f)AVS.

To investigate this, I slightly changed the *move focus* and the *attentional switch* extensions to integrate a weighting of the functional part of the landmark. Doing this, I provide investigations going beyond those presented in Kluth and Schultheis (submitted). The *focus only at function* extension was not further considered, due to its inability to easily weight the functional strength and its weak results so far.

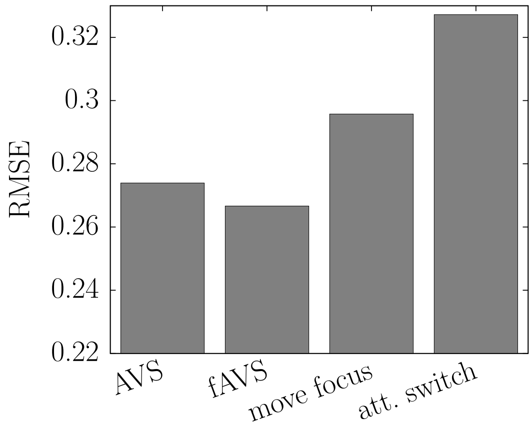
The formula for computing mf in the *move focus* extension (equation (4.8)) is replaced by:

$$mf = \begin{cases} f + w \cdot \overrightarrow{f, ltf} & \text{if } f \text{ is to the left of the functional part} \\ f + w \cdot \overrightarrow{f, rtf} & \text{if } f \text{ is to the right of the functional part} \\ f & \text{if } f \text{ lies in the functional part} \end{cases} \tag{6.1}$$

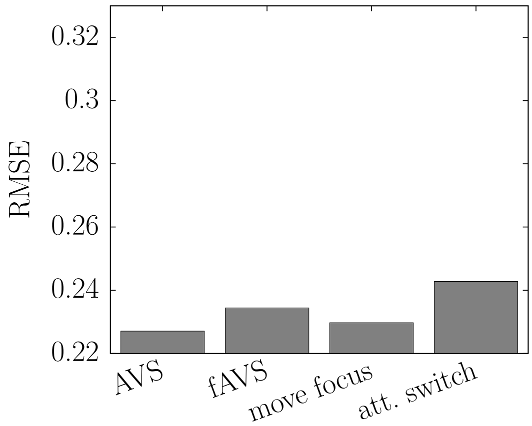
The *attentional switch* extension is changed in its last step. In the modified version the deviations obtained from the geometric focus and the functional focus are combined, but the functional deviation is weighted as follows:

$$angle = \frac{w \cdot funcDev + geomDev}{w + 1} \tag{6.2}$$

In both modified version w is defined as $w = \frac{\varphi}{2}$ to obtain values for φ that are comparable with the values proposed in Carlson et al. (2006), see Section 4.1.2. If $\varphi = 0$ the two modified extensions are the same as AVS – i.e. equivalent to no functional interaction. If $\varphi = 2$ the modified *move focus* extension is the same as the *focus only at*



(a) GOF results for all positions using modified models of *move focus* and *attentional switch*



(b) GOF results for the functional subset using modified models of *move focus* and *attentional switch*

Figure 6.5: GOF results for Carlson-Radvansky et al. (1999) using modified models of *move focus* and *attentional switch*

function extension and the modified *attentional switch* extension acts like the unmodified *attentional switch* extension where function and geometry both play an equal role. For values between 0 and 2 the functional strength is weighted accordingly.

In the following, I will present results for these two modified models, both GOF and SHO, and compare them with the already obtained results for (f)AVS.

6.4.1 Goodness of Fit: Carlson-Radvansky et al.

The GOF values for the modified models on the data from Carlson-Radvansky et al. (1999) are visualized in Figure 6.5. The GOF values for the (f)AVS models are the same as already discussed and visualized for convenience[1]. As expected, the modified versions of *move focus* and *attentional switch* result in lower GOF values than their unmodified versions (cf. Figure 6.1). However, considering all positions, fAVS still results in the best fit. Nevertheless, the modified models achieve GOF values close to fAVS.

For the functional subset the differences in the GOF values of the four models get even more close. Here, the modified *move focus* extension fits better than fAVS. in both data sets, the modified *attentional switch* extension fits comparably bad.

Note, that all three considered extensions can act like the AVS model by using $\varphi = 0$. Consequently, all models are able to provide a fit at least as well as AVS using the best parameters for AVS and setting φ to 0. The reason why the simulated annealing method does not find this optimum might be the bigger parameter space that needs to be searched and that this optimum lies on a parameter space boundary ($\varphi \in [0, 2]$). The SHO method overcomes this problem.

[1] The same is true for subsequent results, both SHO and GOF.

Table 6.3: Best parameters to fit the functional subset corresponding to Figure 6.5b

Param.	AVS	fAVS	move focus	att. switch
λ, att. width	0.2880	0.3980	0.2816	0.3222
φ, func. strength	–	1.3485	1.3467	0.2671
slope	-0.0076	-0.0076	-0.0076	-0.0067
intercept	0.9541	0.9794	0.9634	0.9794
highgain	4.8500	1.5343	3.5357	6.6125

Parameter values

The parameter values leading to the plotted GOF values for the functional subset in Figure 6.5b are listed in Table 6.3. As already discussed above, the effect of the functional parts in the functional subset seems to be of more importance in comparison to all positions as is evident by the best value for φ in fAVS: For the functional subset this value is 1.3485, for all positions the value is only 0.6804 (cf. Table 6.1).

Accordingly, the modified *move focus* extension also supposes a strong functional interaction – interestingly it assumes almost the same value as fAVS ($\varphi = 1.3467$). However, the modified *attentional switch* extension assumes a weak functional interaction ($\varphi = 0.2671$), but also results in the worst fit.

6.4.2 Goodness of Fit: Hörberg

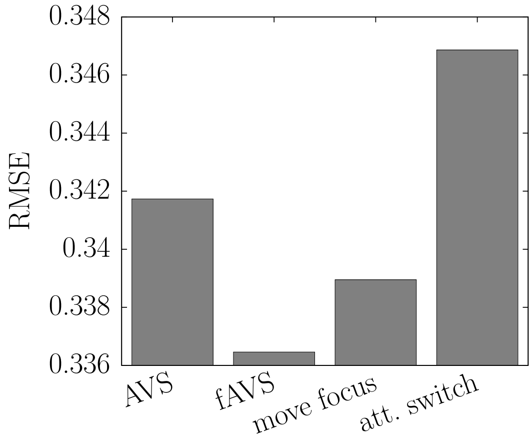
The GOF values for the two data sets from Hörberg (2008) can be found in Figure 6.6. Again, the two modified extensions are able to fit the data better than their unmodified ancestors (cf. Figure 6.2). For the piggy bank data, fAVS gives the best fit and the modified *attentional switch* extension gives the worst fit. Both, fAVS and the modified *move focus* extension, are able to fit the data better than AVS.

Once again, the petrol canister data results in comparably worse fits than the other data sets. However, the modified *attentional switch* extension again provides the worst fit.

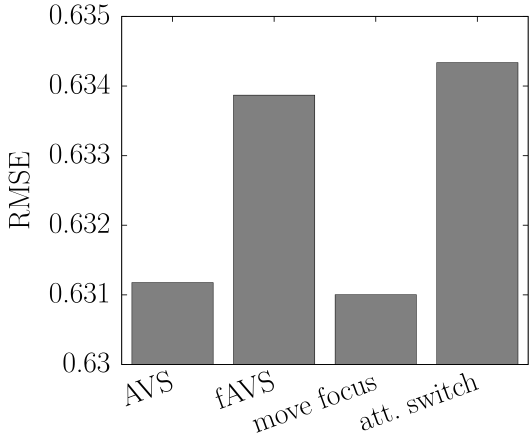
Parameter values

Table 6.4 shows the best parameters of the models to fit the piggy bank data from Hörberg (2008). The size of the functional strength suggested by the models is widespread: from very weak functional interaction in the fAVS model ($\varphi = 0.2805$) to quite strong functional interaction in the modified *move focus* extension ($\varphi = 1.9018$). Since both, the fAVS model and the modified *move focus* extension, fit better than AVS it remains unclear, how strong the functional interaction really is.

Altogether, the just presented GOF values for both data-sets call the results obtained by the unmodified extensions into question. Considering the GOF values, the two modified extensions are able to mirror the data equally well compared to (f)AVS. One may



(a) GOF results for piggy bank using modified models of *move focus* and *attentional switch*

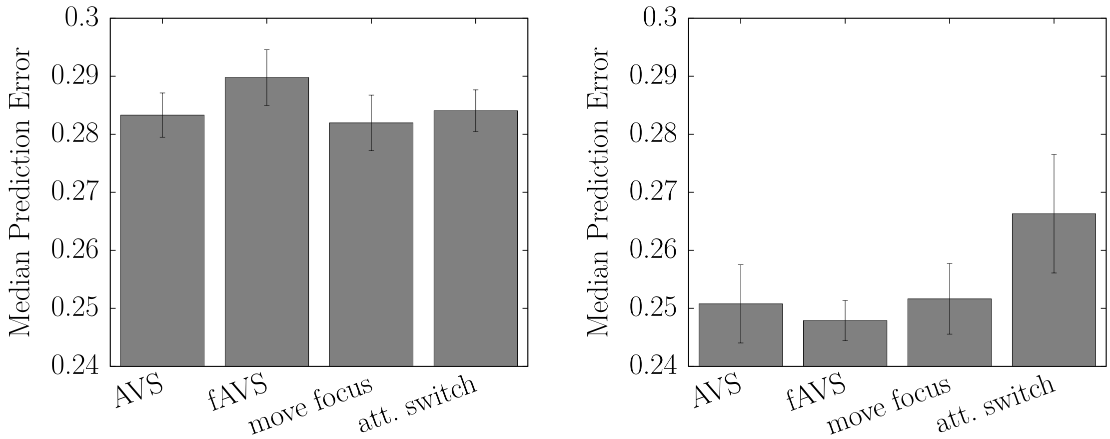


(b) GOF results for petrol canister using modified models of *move focus* and *attentional switch*

Figure 6.6: GOF results for Hörberg (2008) using modified models of *move focus* and *attentional switch*

Table 6.4: Best parameters to fit the piggy bank corresponding to Figure 6.6a

Param.	AVS	fAVS	move focus	att. switch
λ, att. width	1.8172	0.5420	0.4856	1.2725
φ, func. strength	–	0.2805	1.9018	0.7602
slope	-0.0112	-0.0125	-0.0126	-0.0108
intercept	0.9350	0.9334	0.9337	0.9271
highgain	6.5879	0.5880	9.9697	6.7389



(a) SHO results for all positions using modified models of *move focus* and *attentional switch*

(b) SHO results for the functional subset using modified models of *move focus* and *attentional switch*

Figure 6.7: SHO results for Carlson-Radvansky et al. (1999) using modified models of *move focus* and *attentional switch*

conclude a slight preference for the modified *move focus* extension over the modified *attentional switch* extension. To provide further insight into the models' possibilities, I will subsequently present the SHO results.

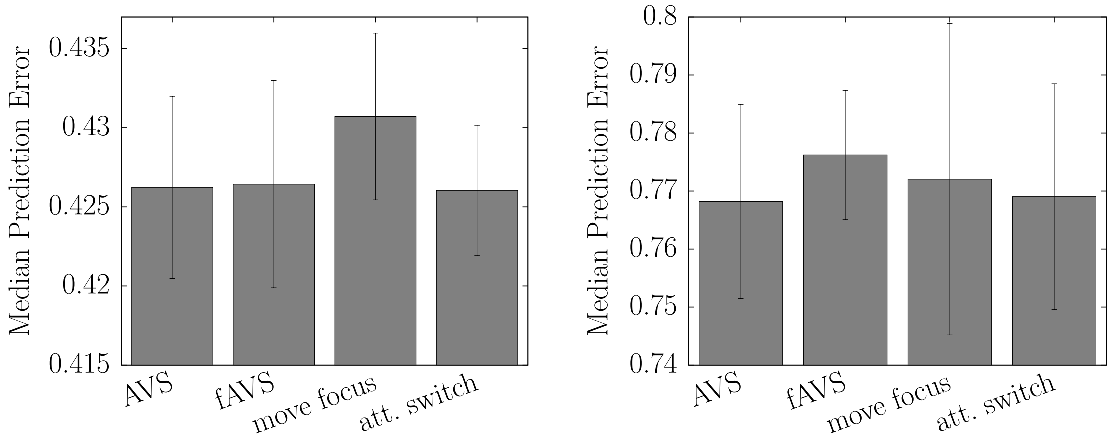
6.4.3 Simple hold-out: Carlson-Radavansky et al.

Comparing the models with the SHO method on the data from Carlson-Radvansky et al. (1999) leads to the results shown in Figure 6.7. Considering all trajector positions, the modified *move focus* extension gives the smallest median prediction error, however, the difference to the other models is very small and all standard errors overlap. The fAVS model and the modified *attentional switch* extension give slightly worse results.

Accordingly, for the the functional subset data no model gives clear evidence for best accounting for the data. Since the functional subset is assumed to contain more functional effects, the very weak preference of the modified *move focus* extension gained from all positions is questionable – even more, since fAVS gives a better SHO result on the functional subset. However, the modified *attentional switch* shows slightly worse behavior than the other models. The fAVS model even outperforms the modified *attentional switch* extension.

6.4.4 Simple hold-out: Hörberg

The SHO results for the Hörberg (2008) data do not provide clear answers, see Figure 6.8. Similar median prediction errors and big standard errors make it impossible to give meaningful interpretations. However, the modified *move focus* extension gives the worst SHO value for the piggy bank.



(a) SHO results for piggy bank using modified models of *move focus* and *attentional switch*

(b) SHO results for petrol canister using modified models of *move focus* and *attentional switch*

Figure 6.8: SHO results for Hörberg (2008) using modified models of *move focus* and *attentional switch*

In the case of the piggy bank, these hard to interpret results might be due to weak functional effects in the data, cf. the value of φ in the fAVS model in Table 6.4 ($\varphi = 0.2805$). However, the modified *move focus* extension assumed strong functional interaction ($\varphi = 1.9018$). The petrol canister already gave hard to interpret results throughout this work.

The comparison with rating data obtained for the preposition *över* might provide more reliable results, since *över* was found to be more influenced by function.

Altogether, the modifications of two of my proposed extensions reject the support for a multimodal attentional distribution concluded from the evaluation of the unmodified extensions. None of the models outperforms all other models. Thus, my results neither support a multimodal nor an unimodal attentional distribution.

7 Conclusion

This work investigated the role of visual spatial attention during spatial term use. The AVS model proposed by Regier and Carlson (2001), the functional extension proposed by Carlson et al. (2006) (fAVS) and three alternative functional extensions (*move focus, focus only at function, attentional switch*) were implemented. The fAVS model assumes split attention; the three alternative functional extensions assume an unimodal distribution of spatial attention. Simulations of these five models on empirical data from Carlson-Radvansky et al. (1999) and Hörberg (2008) were conducted to evaluate the models.

In their first formulation the three alternative extensions proposed in this work were not able to account well for the data, thus lending support to the assumption of a multimodal attentional distribution during spatial term use.

However, modifications of two proposed extensions making them able to account gradually for functional relationships between landmark and trajector led to different results. According to these results, one cannot say whether spatial attention is divisable or not.

Interestingly, the AVS model is able to mirror all considered data, even though it does not take any functional aspects into account. This leads to the conclusion that either the functional effects in the considered data are too small and thus, the functional extensions cannot come into play, or none of the functional extensions capture the mechanisms appropriately which Coventry and colleagues call *extra-geometric*.

To further investigate this, new experiments aiming to reveal stronger functional effects should be designed and conducted. Also, the possibility of using empirical data from the studies of Coventry and colleagues for further model assessment should be explored. Especially, rating data for prepositions which are found to be influenced to a greater extent by function (*over* or *över*) might help.

An important step toward improving the AVS model (and thus its extensions) would be to find a reasonable way to model the shape and functional parts of the trajector, since these aspects are missing in the model. For example, the petrol canister data used in this work cannot be captured from the AVS model or any of its extensions, presumably because of the lack of an appropriate trajector representation (see also the discussion in Hörberg (2008)). Furthermore, Burigo and Sacchi (2013) showed that pure geometric properties of the trajector also affect spatial language apprehension.

Coventry et al. (2010) conducted a study in which they tracked eye movements of people during the apprehension of spatial terms. This study might give valuable data to modify the discussed functional extensions or propose additional extensions to the AVS model assuming attentional distributions found by Coventry et al. (2010).

Another recent study in which eye movements during spatial language comprehension were recorded is reported in Burigo and Knoeferle (2011). This study might as well provide useful empirical data to improve and evaluate the AVS model and its extensions.

Finally, it could be promising to refine the definition of visual attention in the AVS model. Since object-based attention could also be involved in spatial term use, it might be worth not only to consider spatial attention, but also to integrate object-based attention into the AVS model.

In this regard, the biased-competition model that combines feature-, object-, and space-based attention (Kravitz & Behrmann, 2011) could be a promising starting point.

A Implementation

The source code and its documentation can be either found on the CD you should have received or at https://bitbucket.org/kluth/avs.

A.1 Build

Only Linux is tested (so far Ubuntu 12.10, Ubuntu 13.04, Fedora 19, Sabayon 14.01). Maybe it will work on other platforms as well. Feel free to try!

A.1.1 Dependencies

This project uses the `CGAL-Library` (https://www.cgal.org). Just install the corresponding CGAL-packages that should come with your distribution. Also, the `GNU Scientific Library` (https://www.gnu.org/software/gsl) is needed. Again, installing the corresponding GSL-packages should be enough. The source code uses some `C++11` routines. It should work with `gcc` $\geq$ 4.7.2. To compile you also need `cmake`. For visualization you need to install `gnuplot`(http://www.gnuplot.info).

A.1.2 Compilation

```
mkdir build
cd build
cmake ..
cmake ..
make
```

`cmake` needs to be invoked two times to append the `c++11`-flags to the compiler-flags.

A.2 Options/Usage

You might want to start with this line:

```
./avs --landmark ../data/toothbrush.poly --functional ../data/toothbrush_bristles.poly
```

Other example files can be found in the `data/` folder. The syntax of the data-files is described in the further documentation that can be found in the `doc/` folder. The following tables list all available options (also available via `./avs -h`):

I/O-options	
Command	**Action**
--landmark, -l 'landmark.file'	Pass a file containing a CGAL polygon. Will be used as landmark.
--functional, -f 'functional.file'	Pass a file containing a CGAL polygon. Will be used as functional part of the landmark. This option can not be used with –data, -d.
--data, -d 'data.file'	Pass a file containing rating results. This results will be used with the trajector positions found in 'data.file_trajectors'. Functional part of the landmark will be searched in 'data.-file_functionalPart'
--reverse-y, -r	If this flag is set, the y-axis will be grow from top to down (like in computer graphics). Otherwise the y-axis will grow from down to top.
--method, -m [1,2,3,4]	Choose a method out of these 5: AVS = 0, fAVS = 1, move Focus = 2, focus only at function = 3, attentional switch = 4}
--output, -o 'output.txt'	specify the file, where the results should be stored
Parameter options	
Command	**Action**
--parameter-estimation, -p	estimate parameters with the metropolis algorithm. Data and trajector positions must be given.
--simple-hold-out, -s	use simple hold out method to compare methods
--lambda [double-value]	set parameter lambda. Default: 1.0
--slope [double-value]	set parameter slope. Default: -0.006
--intercept [double-value]	set parameter intercept. Default: 1.007
--highgain [double-value]	set parameter highgain. Default: 0.131
--phi [double-value]	the set parameter phi. Only gets considered with method 1. Default: 2.0
Visualization options	
Command	**Action**
--single-trajector, -t x,y	A single trajector position and it's focus to be visualized.
--visualize-vectors, -v	Visualize almost all weighted vectors. Only considered if –single-trajector is set.
--visualize-attention, -a	Visualize attention on the landmark. Only considered if –single-trajector is set.
--color, -c	visualize with colors
--latex	use gnuplots epslatex terminal. Output will be saved in img/figure.tex and img/figure.eps.
--scale, -z scalefactor	the landmark and the functional-part are scaled with this factor. Default: 1.0
--width, -x lowestX:highestX	specify the width limits to compute
--height, -y lowestY:highestY	specify the height limits to compute
--help, -h	print this help message and exit

A.3 License

Bibliography

Borji, A., & Itti, L. (2013, January). State-of-the-art in visual attention modeling. *IEEE transactions on pattern analysis and machine intelligence*, *35*(1), 185–207. doi: 10.1109/TPAMI.2012.89

Broadbent, D. E. (1958). *Perception and communication.* Oxford: Oxford University Press.

Burigo, M., & Knoeferle, P. (2011). Visual attention during spatial language comprehension: Is a referential linking hypothesis enough? *Proceeding of the 33rd Annual Meeting of the Cognitive Science Society*, 919–924.

Burigo, M., & Sacchi, S. (2013, May). Object Orientation Affects Spatial Language Comprehension. *Cognitive science*, *37*(8), 1–22. doi: 10.1111/cogs.12041

Carlson, L. A., & Logan, G. D. (2005). Attention and Spatial Language. In L. Itti, G. Rees, & J. K. Tsotsos (Eds.), *Neurobiology of attention* (pp. 330–336). Amsterdam: Elsevier Academic Press.

Carlson, L. A., Regier, T., Lopez, W., & Corrigan, B. (2006, December). Attention Unites Form and Function in Spatial Language. *Spatial Cognition & Computation*, *6*(4), 295–308. doi: 10.1207/s15427633scc0604_1

Carlson-Radvansky, L. A., Covey, E. S., & Lattanzi, K. M. (1999, November). "What" Effects on "Where": Functional Influences on Spatial Relations. *Psychological Science*, *10*(6), 516–521. doi: 10.1111/1467-9280.00198

Carlson-Radvansky, L. A., & Radvansky, G. A. (1996, January). The Influence of Functional Relations on Spatial Term Selection. *Psychological Science*, *7*(1), 56–60. doi: 10.1111/j.1467-9280.1996.tb00667.x

Carrasco, M. (2011, July). Visual attention: the past 25 years. *Vision research*, *51*(13), 1484–525. doi: 10.1016/j.visres.2011.04.012

Cave, K. R., Bush, W. S., & Taylor, T. G. G. (2010, April). Split attention as part of a flexible attentional system for complex scenes: comment on Jans, Peters, and De Weerd (2010). *Psychological review*, *117*(2), 685–96. doi: 10.1037/a0019083

Chen, Z. (2012, July). Object-based attention: a tutorial review. *Attention, perception & psychophysics*, *74*(5), 784–802. doi: 10.3758/s13414-012-0322-z

Chun, M. M., Golomb, J. D., & Turk-Browne, N. B. (2011, January). A taxonomy of external and internal attention. *Annual review of psychology*, *62*, 73–101. doi: 10.1146/annurev.psych.093008.100427

Coventry, K. R., Cangelosi, A., Rajapakse, R., Bacon, A., Newstead, S., Joyce, D., & Richards, L. V. (2005). Spatial prepositions and vague quantifiers: Implementing the functional geometric framework. In C. Freksa, M. Knauff, B. Krieg-Brückner, B. Nebel, & T. Barkowsky (Eds.), *Spatial Cognition IV. Reasoning, Action, Interaction* (pp. 98–110). Springer Berlin Heidelberg. doi: 10.1007/978-3-540-32255-9

Coventry, K. R., Carmichael, R., & Garrod, S. C. (1994). Spatial Prepositions, Object-Specific Function, and Task Requirements. *Journal of Semantics*, *11*(4), 289–309. doi: 10.1093/jos/11.4.289

Coventry, K. R., & Garrod, S. C. (2004). *Saying, Seeing and Acting: The Psychological Semantics of Spatial Prepositions.* New York: Taylor & Francis.

Coventry, K. R., & Guijarro-Fuentes, P. (2004). Las preposiciones en español y en inglés: la importancia relativa del espacio y función. *Cognitiva.*

Coventry, K. R., Lynott, D., Cangelosi, A., Monrouxe, L., Joyce, D., & Richardson, D. C. (2010, March). Spatial language, visual attention, and perceptual simulation. *Brain and language*, *112*(3), 202–13. doi: 10.1016/j.bandl.2009.06.001

Coventry, K. R., Prat-Sala, M., & Richards, L. (2001, April). The Interplay between Geometry and Function in the Comprehension of Over, Under, Above, and Below. *Journal of Memory and Language*, *44*(3), 376–398. doi: 10.1006/jmla.2000.2742

Desimone, R., & Duncan, J. (1995, January). Neural mechanisms of selective visual attention. *Annual review of neuroscience*, *18*, 193–222. doi: 10.1146/annurev.ne.18.030195.001205

Deutsch, J. A., & Deutsch, D. (1963). Attention: Some Theoretical Considerations. *Psychological Review*, *70*(1), 80–90. doi: 10.1037/h0039515

Downing, C. J., & Pinker, S. (1985). The Spatial Structure of Visual Attention. In M. I. Posner & O. Martin (Eds.), *Attention and Performance, XI* (pp. 171–187). Hillsdale, N.J.: Lawrence Erlbaum Associates Inc.

Efron, B., & Tibshirani, R. J. (1993). *An Introduction to the Bootstrap.* New York: Chapman & Hall.

Egly, R., Driver, J., & Rafal, R. D. (1994, June). Shifting visual attention between objects and locations: evidence from normal and parietal lesion subjects. *Journal of experimental psychology. General*, *123*(2), 161–77.

Eriksen, C. W., & Yeh, Y.-Y. (1985, October). Allocation of attention in the visual field. *Journal of experimental psychology. Human perception and performance*, *11*(5), 583–97.

Feng, J., & Spence, I. (2013). A mixture distribution of spatial attention. *Experimental psychology*, *60*(3), 149–56. doi: 10.1027/1618-3169/a000182

Frintrop, S., Rome, E., & Christensen, H. I. (2010, January). Computational Visual Attention Systems and their Cognitive Foundations. *ACM Transactions on Applied Perception*, *7*(1), 1–39. doi: 10.1145/1658349.1658355

Georgopoulos, A. P., Schwartz, A. B., & Kettner, R. E. (1986). Neuronal population coding of movement direction. *Science*, *233*(4771), 1416–1419.

Hagendorf, D. H., Krummenacher, P. D. J., Müller, P. D. H.-J., & Schubert, P. D. T. (2008). Selektive Aufmerksamkeit. In *Wahrnehmung und Aufmerksamkeit* (pp. 179–201). Berlin Heidelberg: Springer.

Hörberg, T. (2006). *Influences of Form and Function on Spatial Relations: Establishing functional and geometric influences on projective prepositions in Swedish.* Magister thesis, Stockholm University.

Hörberg, T. (2008, September). Influences of Form and Function on the Acceptability of Projective Prepositions in Swedish. *Spatial Cognition & Computation*, *8*(3), 193–218. doi: 10.1080/13875860801993652

James, W. (1890). *The principles of psychology* (Vol. 1). New York: Henry Holt and Co, Inc. Retrieved March 13, 2014, from https://openlibrary.org/books/OL7112992M/The_principles_of_psychology

Jans, B., Peters, J. C., & De Weerd, P. (2010, April). Visual Spatial Attention to Multiple Locations at Once: The Jury Is Still Out. *Psychological review*, *117*(2), 637–84. doi: 10.1037/a0019082

Jefferies, L. N., Enns, J. T., & Di Lollo, V. (2013). The Flexible Focus: Whether Spatial Attention Is Unitary or Divided Depends on Observer Goals. *Journal of experimental psychology. Human perception and performance*, *39*(6), 1–7. doi: 10.1037/a0034734

Kluth, T., & Schultheis, H. (submitted). *Attentional Distribution and Spatial Language.* (Submitted to Spatial Cognition 2014)

Kravitz, D. J., & Behrmann, M. (2011, November). Space-, object-, and feature-based attention interact to organize visual scenes. *Attention, perception & psychophysics*, *73*(8), 2434–47. doi: 10.3758/s13414-011-0201-z

Landau, B., & Jackendoff, R. (1993). "What" and "where" in spatial language and spatial cognition. *Behavioral and Brain Sciences*, *16*, 217–265.

Lee, C., Rohrer, W. H., & Sparks, D. L. (1988, March). Population coding of saccadic eye movements by neurons in the superior colliculus. *Nature*, *332*(6162), 357–60. doi: 10.1038/332357a0

Lin, E. L., & Murphy, G. L. (1997). Effects of background knowledge on object categorization and part detection. *Journal of Experimental Psychology: Human Perception and Performance*, *23*(4), 1153–1169. doi: 10.1037//0096-1523.23.4.1153

Lipinski, J., Schneegans, S., Sandamirskaya, Y., Spencer, J. P., & Schöner, G. (2012, November). A neurobehavioral model of flexible spatial language behaviors. *Journal of experimental psychology. Learning, memory, and cognition*, *38*(6), 1490–511. doi: 10.1037/a0022643

Logan, G. D., & Sadler, D. D. (1996). A computational analysis of the apprehension of spatial relations. In P. Bloom, M. A. Peterson, L. Nadel, & M. F. Garrett (Eds.), *Language and space. language, speech, and communication.* (pp. 493–529). Cambridge, MA: M.I.T. Press.

Madras, N. (2002). Markov Chain Monte Carlo. In *Lectures on monte carlo methods* (pp. 53–82). Providence, Rhode Island: American Mathematical Society.

Metropolis, N., Rosenbluth, A. W., Rosenbluth, M. N., Teller, A. H., & Teller, E. (1953). Equation of state calculations by fast computing machines. *The journal of chemical physics*, *21*(6), 1087–1092.

Mole, C. (2013). Attention. In E. N. Zalta (Ed.), *The stanford encyclopedia of philosophy* (Fall 2013 ed.). Retrieved March 13, 2014, from http://plato.stanford.edu/archives/fall2013/entries/attention/

Pitt, M. A., & Myung, I. J. (2002). When a good fit can be bad. *Trends in cognitive sciences*, *6*(10), 421–425.

Posner, M. I. (1980, February). Orienting of attention. *Quarterly journal of experimental psychology*, *32*(1), 3–25.

Regier, T., Carlson, L., & Corrigan, B. (2005). Attention in Spatial Language: Bridging Geometry and Function. In L. Carlson & E. van der Zee (Eds.), *Functional features in language and space* (pp. 191–204). New York: Oxford University Press.

Regier, T., & Carlson, L. A. (2001). Grounding Spatial Language in Perception: An Empirical and Computational Investigation. *Journal of Experimental Psychology: General*, *130*(2), 273–298. doi: 10.1037//0096-3445.130.2.273

Roberts, S., & Pashler, H. (2000). How persuasive is a good fit? A comment on theory testing. *Psychological review*, *107*(2), 358–367. doi: 10.1037//0033-295X.1O7.2.358

Scholl, B. J. (2001, June). Objects and attention: the state of the art. *Cognition*, *80*(1-2), 1–46.

Schultheis, H., Singhaniya, A., & Chaplot, D. S. (2013). Comparing Model Comparison Methods. In *Proceedings of the 35th annual conference of the cognitive science society.*

Soto, D., & Blanco, M. J. (2004, January). Spatial attention and object-based attention: a comparison within a single task. *Vision Research*, *44*(1), 69–81. doi: 10.1016/j.visres.2003.08.013

Vecera, S. P., & Behrmann, M. (2001). Attention and unit formation: A biased competition account of object-based attention. In T. F. Shipley & P. J. Kellman (Eds.), *From fragments to objects segmentation and grouping in vision* (Vol. 130, pp. 145–180). North-Holland. doi: 10.1016/S0166-4115(01)80026-1

Wilson, H. R., & Kim, J. (1994, July). Perceived motion in the vector sum direction. *Vision research*, *34*(14), 1835–42.

Eidesstattliche Erklärung

Hiermit versichere ich, dass ich die vorliegende Arbeit mit dem Titel „*One Focus or Many? – Modeling Attentional Distributions During Spatial Term Use*“ selbständig verfasst habe, dass ich keine anderen Quellen und Hilfsmittel als die angegebenen benutzt habe und dass die Stellen der Arbeit, die anderen Werken – auch elektronischen Medien – dem Wortlaut oder Sinn nach entnommen wurden, auf jeden Fall unter Angabe der Quelle als Entlehnung kenntlich gemacht worden sind.

Ort, Datum	Thomas Kluth