H22: Which of the following (when stationary) are reversible Markov chains?

a) [4pts] The chain \( X = (X_n; n \geq 0) \) having transition matrix:

\[
P = \begin{pmatrix}
1 - \alpha & \alpha \\
\beta & 1 - \beta
\end{pmatrix},
\]

where \( \alpha + \beta > 0 \). (Note: The result can depend on the values of \( \alpha \) and \( \beta \).)

b) [4pts] The chain \( X = (X_n; n \geq 0) \) having transition matrix:

\[
P = \begin{pmatrix}
0 & p & 1 - p \\
1 - p & 0 & p \\
p & 1 - p & 0
\end{pmatrix},
\]

where \( 0 < p < 1 \). (Note: The result can depend on the values of \( p \).)

H23: [3pts] Show that every Markov chain with a symmetric transition matrix and finite state space is reversible.

H24: Consider two players, \( A \) and \( B \), playing the following tossing coin game: at the beginning the player \( A \) has 100 euros and the player \( B \) has 200 euros. At each time step the players toss a coin with probability \( p \in [0, 1] \) of head. If the result is a head the player \( A \) gives 1 euro to the player \( B \), otherwise is the player \( B \) who gives 1 euro to the player \( A \). The game stops when the fortune of one of the players reaches 0 euros. We call \( X_n \) the fortune of the player \( A \) at time \( n \).

a) [3pts] Is the process \( (X_n)_{n \geq 1} \) a Markov chain? If yes, give the state space and the corresponding transition matrix.

b) [6pts] For \( p = 0.5 \), \( p = 0.2 \) and \( p = 0.7 \), simulate 5 times the Markov chain \( (X_n)_{n \geq 1} \) till the first time \( X_n \) is equal to 0 or 300 euros. Provide the results in a graph: you can either provide one graph for each simulation (5 graphs for each value of \( p \)) or you can group your results providing on each graph one simulation for each value of \( p \) (5 graphs in total).