Color Models

- Used to describe color as accurately as possible.
- Uses the fact that colors can be described by combinations of three basic colors, called primary colors.

**CIE (Commission International de l’Eclairage - International Color Commision)** organisation produced two models for defining color:

- 1931: Measured on 10 subjects (!) on samples subtending 2 (!) degrees of the field of view
- 1964: Measured on larger number of subjects subtending 10 degrees of field of view

- The CIE 1931 model is the most commonly used
  - It defines three primary “colors” X, Y and Z that can be used to describe all visible colors, as well as a standard white, called C.
  - The range of colors that can be described by combinations of other colors is called a color gamut.

- Since it is impossible to find three colors with a gamut containing all visible colors, the CIE’s three primary colors are imaginary. They cannot be seen, but they can be used to define other visible colors.

Doing the Experiment

- People sit in a dark room matching colors:

- But any three R, G and B can’t match all colors... (for reasons we'll be exploring soon):

- Sometimes need to add some R to the sample you are trying to match: (Expressed mathematically as “-R”)
CIE Space for Color Matching

- Defined X, Y, and Z primaries to replace red, green and blue primaries
- $x_\lambda$, $y_\lambda$, and $z_\lambda$, color matching functions for these primaries
- Y chosen so that $y_\lambda$ matches luminous efficiency function
- $x_\lambda$, $y_\lambda$, and $z_\lambda$ are linear combinations of $r_\lambda$, $g_\lambda$, and $b_\lambda$

$$\Rightarrow \text{RGB}_i \leftrightarrow \text{XYZ}_i \text{ via a matrix}$$

Mapping the mathematical color matching functions to $x_\lambda$, $y_\lambda$, and $z_\lambda$ for the 1931 CIE X, Y, and Z primaries. They are defined tabularly at 1 nm intervals for color samples that subtend 2° field of view on retina

CIE 1931 Model

- To define a color in CIE model, provide weights for the X, Y and Z primaries, just as you would for an RGB display (e.g. color = $xX + yY + zZ$).
- X, Y and Z form a three dimensional color volume.
- We can ignore the dimension of luminance by normalizing with total light intensity, $x+y+z = 1$.
  This gives chromaticity values:
  $$x' = x/(x+y+z)$$
  $$y' = y/(x+y+z)$$
  $$z' = 1 - x' - y'$$

- Plotting $x'$ and $y'$ gives the CIE chromaticity diagram.
- Color gamuts are found by taking the convex hull of the primary colors.
- Complements are found by inscribing a line from the color through C to the edge of the diagram.
CIE 1931 Model

• Hue of a color:
  • found by inscribing a line from C (white) through the color to the edge of the diagram. The hue is the wavelength of the color at the intersection of the edge and the line.

• Saturation of a color:
  • found by taking the ratio of the distance of the color from C on the above line and the length of the whole line.

• Complementary colors
  • can be mixed to produce white light (a non-spectral color!)

• White
  • can be produced by (approx) constant spectral distribution as well as by only two complementary colors, e.g., greenish-blue, \( D \), and reddish-orange, \( E \).

• Some nonspectral colors
  • (colors not on spectral locus, like \( G \)) cannot be defined by dominant wavelength; defined by complementary dominant wavelength.

Color gamuts

• Colors add linearly in CIE: All mixture of \( I \) and \( J \) lie on the line connecting them.

• Thus, all possible mixtures of \( I \), \( J \) and any third color, \( K \), (or additional colors) lie within their convex hull. Called the color gamut.

• No finite number of primaries can include all visible colors!

➢ Device dependent gamut: Mapping colors must be done carefully!
Why the Chromaticity Diagram is not triangular

- No gamut described by a linear combination of $n$ physical (real) primaries (yielding a convex hull) can simulate the eye’s responses to all visible colors
- Review of CIE:
  - shape of CIE space determined by Matching Experiment:
    - subject shown color and asked to create metameric match from colored monochromatic primaries, R, G and B
  - most colors can be matched, some can’t because of way response curves overlap
  - would need “negative amounts” of some primary to match all visible color samples;
    - not physically possible, but can be simulated by adding that color to sample to be matched.
  - to simplify, CIE primaries $X$, $Y$, and $Z$ used to get all positive color matching functions

Chromatic Opponent Channels on Chromaticity Diagram

- Let’s see in another way why 3 (indeed $n$) physical primaries aren’t sufficient to match an arbitrary color by looking at response function.

Why would red-green and blue-yellow be useful axes to specify color with?
Color Models for Raster Graphics

- Purpose: specify colors in some gamut
- Gamut is a subset of all visible chromaticities so model does not contain all visible colors
- 3D color coordinate system subset containing all colors within a gamut
- Means to convert to other model(s)
- Example color model: RGB
  - 3D Cartesian coordinate system
  - unit cube subset
  - Use CIE XYZ space to convert to and from all other models

Color Models for Raster Graphics

- Hardware-oriented models: not intuitive – do not relate to concepts of hue, saturation, brightness
  - RGB, used with color CRT monitors
  - YIQ, broadcast TV color system
  - CMY (cyan, magenta, yellow) color printing
  - CMYK (cyan, magenta, yellow, black) color printing
  - IRODORI, six-primary-color projection system

- User-oriented models
  - HSV (hue, saturation, value)
  - also called HSB (B for brightness)
  - HLS (hue, lightness, saturation)
  - The Munsell system
  - CIE Lab
The RGB Color Model

- RGB primaries are additive:
  - Main diagonal => gray levels
    - black is (0, 0, 0)
    - white is (1, 1, 1)
  - Hue is defined by the one or two largest parameters
  - Saturation can be controlled by varying the collective minimum value of R, G and B
  - Luminance can be controlled by varying magnitudes while keeping ratios constant

The RGB Color Model

- Conversion from one RGB gamut to another
  - convert one to XYZ, then convert from XYZ to another
- Form of each transformation:
  \[
  \begin{bmatrix}
  X \\
  Y \\
  Z \\
  \end{bmatrix} = \begin{bmatrix}
  X_r & X_g & X_b \\
  Y_r & Y_g & Y_b \\
  Z_r & Z_g & Z_b \\
  \end{bmatrix} \begin{bmatrix}
  R \\
  G \\
  B \\
  \end{bmatrix}
  \]
  - Where \(X_r, X_g,\) and \(X_b\) are weights applied to monitor’s RGB colors to find \(X,\) etc.
  - \(M\) is 3 x 3 matrix of color-matching coefficients
    \[
    \begin{bmatrix}
    X \\
    Y \\
    Z \\
    \end{bmatrix} = \begin{bmatrix}
    R \\
    G \\
    B \\
    \end{bmatrix}
    \]
    - Let \(M_1\) and \(M_2\) be matrices to convert from two monitor’s gamuts to CIE
    - \(M_2^{-1} \ M_1\) converts from RGB of monitor 1 to RGB of monitor 2
The RGB Color Model

- But what if…
  - \( C_1 \) in gamut of monitor 1 but not in gamut of monitor 2, i.e., \( C_2 = M_2^{-1} M_1 C_1 \) outside unit cube and hence not displayable?

- **Solution 1**: clamp RGB at 0 and 1
  - simple, but distorts color relations

- **Solution 2**: compress gamut on monitor 1 by scaling all colors from monitor 1 toward center of gamut 1
  - ensure that all displayed colors on monitor 1 map onto monitor 2

Additive vs. Subtractive Color Systems

- Print systems use subtractive color system:
  - RGB: Red, Green, Blue
  - CMY: Cyan, Magenta, Yellow
The CMY(K) Color Model

- Used in electrostatic/ink-jet plotters that deposit pigment on paper
- Cyan, magenta, and yellow are complements of red, green, and blue
- **Subtractive primaries**: colors are specified by what is subtracted from white light, rather than by what is added to blackness
- Cartesian coordinate system
- Subset is unit cube
  - white is at origin, black at (1, 1, 1):

  \[
  \begin{bmatrix}
  C \\ M \\ Y 
  \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} R \\ G \\ B \end{bmatrix}
  \]

The CMY(K) Color Model

- Color printing presses, some color printers use CMYK (K=black)
- K used instead of equal amounts of CMY
  - called **undercolor removal**
  - richer black
  - less ink deposited on paper – dries more quickly
- First approximation – nonlinearities must be accommodated:
  - \( K = \min(C, M, Y) \)
  - \( C' = C - K \)
  - \( M' = M - K \)
  - \( Y' = Y - K \)
The YIQ Color Model

- Recoded RGB for transmission efficiency, compatibility with B/W broadcast TV;
- used for NTSC (National Television Standards Committee (cynically, “Never The Same Color”))
- \( Y = \text{CIE’s } Y \) (luminance); I and Q encode chromaticity
- Only \( Y = 0.3R + 0.59G + 0.11B \) shown on B/W monitors:
  \[
  \begin{bmatrix}
  Y \\
  I \\
  Q
  \end{bmatrix} =
  \begin{bmatrix}
  0.30 & 0.59 & 0.11 \\
  0.60 & -0.28 & -0.32 \\
  0.21 & -0.52 & 0.31
  \end{bmatrix}
  \begin{bmatrix}
  R \\
  G \\
  B
  \end{bmatrix}
  \]
  - Weights: relative brightness of each primary
  - assumes white point is illuminant C:
    \( x_w = 0.31, y_w = 0.316, \) and \( Y_w = 100.0 \)
- Preparing color material which may be seen on B/W broadcast TV, adjacent colors should have different \( Y \) values
- NTSC encoding of YIQ:
  - 4 MHz \( Y \) (eye most sensitive to \( \triangle \) luminance)
  - 1.5 MHz \( I \) (small images need 1 color dimension)
  - 0.6 MHz \( Q \)

The HSV Color Model

- Hue, saturation, value (brightness)
- Hexcone subset of cylindrical (polar) coordinate system
- Single hexcone HSV color model.
  (The \( V = 1 \) plane contains the RGB model’s \( R = 1, G = 1, B = 1, \) in the regions shown):
  - The eye can see
    - about 128 different hues, and
    - about 130 different saturations.
    - The number of values varies between 16 (blue) and 23 (yellow)
  - Has intuitive appeal of the artist’s tint, shade, and tone model
    - pure red = \( H = 0, S = 1, V = 1 \); pure pigments are \( (I, 1, 1) \)
    - tints: adding white pigment \( \leftrightarrow \) decreasing \( S \) at constant \( V \)
    - shades: adding black pigment \( \leftrightarrow \) decreasing \( V \) at constant \( S \)
    - tones: decreasing \( S \) and \( V \)
The HSV Color Model

- Colors on $V = 1$ plane are not equally bright
- Complementary colors $180^\circ$ opposite
- Saturation measured relative to color gamut represented by model which is subset of chromaticity diagram:
  - therefore, 100% $S$ 100% excitation purity
- Top of HSV hexcone is projection seen by looking along principal diagonal of RGB color
- RGB subcubes are plane of constant $V$
- Code for RGB ↔ HSV in (Foley et al., 1996, pp. 592, 593)
- Note:
  - linear path RGB linear path in HSV!

The HLS color Model

- Hue, lightness, saturation
- Double-hexcone subset
- Maximally saturated hues are at $S = 1$, $L = 0.5$
- Less attractive for sliders or dials
- Neither $V$ nor $L$ correspond to $Y$ in YIQ!
- Conceptually easier for some people to view white as a point
Problems with Standard Color Systems

- They are perceptually non-uniform
  - move through color space from color $C_1$ to a new color $C_1'$ through a distance $\Delta C$
    \[ C_1' = C_1 + \Delta C \]
  - move through the same distance $\Delta C$, starting from a different color $C_2$
    \[ C_2' = C_2 + \Delta C \]
  - the change in color in both cases is mathematically equal, but is not perceived as equal!

- Interpolation, e.g., by moving a slider almost always causes a perceptual change in the other two parameters, which is not reflected by changes in those sliders; thus, changing hue frequently will affect saturation and value (even in Photoshop)!

- Ideally want a perceptually uniform space:
  - two colors that are equally distant are perceived as equally distant, and changing one parameter does not perceptually alter the other two

- Historically, the first perceptually-uniform color space was the Munsell system

The Munsell System

- Created from perceptual data, not as a transformation of or approximation to CIE
- Uniform perceptually-based 3D space
  - accounts for the fact that a bright yellow is much lighter than a bright blue, and that many more levels of saturation of blue can be distinguished than of yellow
- Magnitude of change in one parameter always maps to the same effect on perception
- Hues arranged on a circle
  - a 20 degree rotation through this circle always causes the same perceptual change, no matter where on the circle you start from
  - does not cause changes in saturation or value
- Saturation as distance from center of circle
  - moving away from the center a certain distance always causes the same perceptual change
  - does not cause changes in hue or value
- Value as height in space
  - moving vertically always causes the same perceptual change
  - does not cause changes in hue or saturation
CIE Lab

- CIE Lab was introduced in 1976
  - popular for use in measuring reflective and transmissive objects
- Three components:
  - L* is luminosity
  - a* is red/green axis
  - b* is yellow/blue axis
- Mathematically described space and a perceptually uniform color space
- Given white = \((X_n, Y_n, Z_n)\)

\[
L^* = 116 \left( \frac{Y}{Y_n} \right)^{1/3} - 16, \text{ when } \frac{Y}{Y_n} > 0.008856 \\
L^* = 903 \cdot 0.292 \left( \frac{Y}{Y_n} \right) \text{ when } \frac{Y}{Y_n} \leq 0.008856 \\
a^* = 500 \left( f \left( \frac{X}{X_n} \right) - f \left( \frac{Z}{Z_n} \right) \right) \\
b^* = 200 \left( f \left( \frac{X}{X_n} \right) - f \left( \frac{Z}{Z_n} \right) \right)
\]

where \( f(t) = t^{1/3} \) when \( \frac{Y}{Y_n} > 0.008856 \)
else \( f(t) = 7.787 t + 16 / 116 \)

- These transformations cause regions of colors perceived as identical to be of the same size

Color Model Pros and Cons

- RGB
  + Cartesian coordinate system
  + linear
  + hardware-based (easy to transform to video)
  + tristimulus-based
  - hard to use to pick and name colors
  - doesn’t cover gamut of perceivable colors
  - non-uniform: equal geometric distance => unequal perceptual distance

- CIE
  + covers gamut of perceived colors
  + based on human perception (matching experiments)
  + linear
  + contains all other spaces
  - non-uniform (but variations such as CIE Lab are closer to Munsell, which is uniform)
  - xy-plot of chromaticity horseshoe diagram doesn’t show luminance
Color Model Pros and Cons

- **(CIE cont.)** Example: Photoshop Lab color model is based on CIE Lab space
  - based on psychological colors (y-b, r-g, w-b)
  - terrible interface (see Photoshop)
    - no visualization of the color space
    - very difficult to determine what values mean if you are unfamiliar with the space
    - picks colors which are out of the print gamut
  - primarily used as an internal space to convert between RGB and CMYK
- **HSV**
  - easy to convert to RGB
  - easy to specify colors
  - nonlinear
  - doesn’t cover gamut of perceivable colors
  - nonuniform